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A deformation of twistor space and a chiral mass term in $N = 4$ super Yang-Mills Theory

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Abstract: Super twistor space admits a certain (super) complex structure deformation that preserves the Poincaré subgroup of the symmetry group $PSL(4|4)$ and depends on 10 parameters. In a previous paper [1], it was proposed that in twistor string theory this deformation corresponds to augmenting $N = 4$ super Yang-Mills theory by a mass term for the left-chirality spinors. In this paper we analyze this proposal in more detail. We calculate 4-particle scattering amplitudes of fermions, gluons and scalars and show that they are supported on holomorphic curves in the deformed twistor space.

Keywords: Superspaces, Topological Strings, Duality in Gauge Field Theories, Topological Field Theories.

Contents

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1. Introduction

Twistor [2] string theory [3] studies perturbative scattering amplitudes of massless particles in $N = 4$ Super-Yang-Mills theory in terms of a topological B-model with target space $\mathbb{CP}^{3|4}$. This target space is a Calabi-Yau supermanifold [4][5]. (For alternative formulations of twistor string theory see [6][7][8].) Twistor techniques are, in general, useful for dealing with massless particles. They have recently been used to derive simple expressions for scattering amplitudes that have previously never been written in closed form. (See [9] for a recent review.)

In this paper we would like to describe a theoretical extension of twistor string theory that includes a mass term for the fermions of the vector multiplet of Super-Yang-Mills theory. Such a mass term, of course, breaks supersymmetry and conformal invariance as well. In general, a mass term precludes the use of the twistor transform which requires that external particles have lightlike momenta. (But see [10] for recent developments that use twistor techniques indirectly to calculate scattering amplitudes of massive particles.) However, if the mass term only involves spinors of one chirality and does not include the spinors of the opposite chirality, the plane-wave solutions of the free Dirac equation are still lightlike. Of course, such a model breaks CPT symmetry, but it is consistent mathematically, and we can calculate scattering amplitudes in this model. The amplitudes are holomorphic functions of the chiral mass parameters.

The physical relevance of the scattering amplitudes that we get in such a model can be described as follows. The scattering amplitudes of a model with a CPT-invariant fermion mass term depend on the complex mass parameter M and its complex conjugate M^*. It can be written as an analytic expression in two formally independent variables M and M^*. The amplitudes of the chiral-mass theory can be defined as the expressions that we get when we formally set $M^* = 0$ in the physical amplitudes.

In this work we study the twistor approach to $N = 4$ Super-Yang-Mills theory with an extra chiral mass term, and we expand on ideas presented in [1]. There, it was argued that the free-field equations of motion of the augmented theory still have a twistor description; the relevant twistor space is a certain super complex structure deformation of $\mathbb{CP}^{3|4}$. In this paper we calculate 4-particle scattering amplitudes and extend the notion of *maximally helicity violating* (MHV) amplitudes to include the chiral mass term. In the massless theory, Witten discovered that MHV scattering amplitudes vanish, when expressed in twistor variables, unless certain algebraic conditions hold. The amplitude does not vanish only if there exists an algebraic curve of degree $d = 1$ in supertwistor space, $\mathbb{CP}^{3|4}$, such that all the twistors that label the external particles lie on this curve [3]. Does a similar assertion hold for the theory with the chiral mass term?

In this paper we will explore this question for 4-particle amplitudes. In section 3 we extend the definition of helicity to the fermions with a chiral mass term, and we calculate 4-particle (extended) MHV scattering amplitudes. In addition to the chiral mass term, we also include in the calculations a possible 3-scalar interaction, which has the same dimension ($\Delta = 3$) and R-symmetry quantum numbers as the fermion mass term. In section 4 we describe the deformation of super twistor space that corresponds to adding the chiral mass term, and we look for algebraic curves in the deformed space. There, we define a natural extension of the notion of degree $d = 1$ curves for the deformed case; the equations describing these curves contain quadratic terms. In section 5 we show that if we set the 3-scalar coupling correctly, 4-particle (extended) MHV amplitudes are indeed supported on these $d = 1$ algebraic curves. Furthermore, we find that the amplitudes are given by an integral over the moduli space of $d = 1$ curves that is essentially the same as the one for the massless case [3]; the only modification is the expression for the curve itself. We conclude with a discussion in section 7. The appendices contain more technical details about the Feynman rules in the presence of the unusual CPT-violating chiral mass terms.

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2. Chiral and anti-chiral fermion mass terms

We denote the negative helicity fermions by ψ_α^A, where α is a spinor index ($\alpha = 1, 2$) and A is an $SU(4)$ R-symmetry index ($A = 1, \ldots, 4$). We denote the positive helicity fermions by $\overline{\psi}_A^{\dot\alpha}$. The full $N = 4$ Super Yang-Mills Lagrangian is presented in appendix B, for completeness. An anti-chiral mass term is $M_{AB}\psi_\alpha^A\psi^{\alpha B}$ and a chiral mass term is $M^{AB}\overline{\psi}_{\dot\alpha A}\overline{\psi}_B^{\dot\alpha}$. Here $M_{AB} = M_{BA}$ and $M^{AB} = M^{BA}$ are the corresponding mass matrices, with 10 independent complex parameters each. We are going to add a chiral mass term to the $N = 4$ SYM Lagrangian. This, of course, breaks CPT invariance, but the perturbative Feynman diagrams are well-defined.

2.1 Free field equations of motion

In the presence of a chiral mass term, the negative helicity fermions acquire a left-chirality ($\dot\alpha$) component. To see this, we write down the Dirac equations:

$$p_{\alpha\dot\alpha}\overline{\psi}_A^{\dot\alpha} = 0, \qquad p^{\alpha\dot\alpha}\psi_\alpha^A = M^{AB}\overline{\psi}_B^{\dot\alpha}. \tag{2.1}$$

These equations imply that the momentum $p_{\alpha\dot\alpha}$ is lightlike. It can therefore be written as a product of two spinors,

$$p_{\alpha\dot\alpha} = \lambda_\alpha\tilde\lambda_{\dot\alpha}, \tag{2.2}$$

as in the $M^{AB} = 0$ case. A basis for the solutions of (2.1) is given by

$$\overline{\psi}_A^{\dot\alpha} = \tilde\lambda^{\dot\alpha}\widetilde{\varrho}_A, \qquad \psi_\alpha^A = \lambda_\alpha\varrho^A + M^{AB}\eta_\alpha\widetilde{\varrho}_B, \tag{2.3}$$

where $\widetilde{\varrho}_A, \varrho^A$ are arbitrary parameters (scalars in the fundamental representation of the R-symmetry group) and η^α is an arbitrary chiral spinor that is only required to satisfy

$$\lambda^\alpha\eta_\alpha = 1. \tag{2.4}$$

Once η_α is fixed, we can define *helicity* as follows: A solution with helicity $(-)$ has $\overline{\psi}_A^{\dot\alpha} = 0$ and $\psi^{\alpha A} = \lambda^\alpha\varrho^A$; a solution with helicity $(+)$ has $\overline{\psi}_A^{\dot\alpha} = \tilde\lambda^{\dot\alpha}\widetilde{\varrho}_A$ and $\psi^{\alpha A} = M^{AB}\eta^\alpha\widetilde{\varrho}_B$.[1] In Feynman diagrams, external lines of negative helicity fermions only have left-moving $\psi^{\alpha A}$ components, but external lines of positive helicity fermions have both left-moving and right-moving components. This is depicted in figure 2 and figure 3.

2.2 3-scalar interaction

The fermion mass term that we added in section 2.1 is a linear combination of operators

$$\mathcal{V}'_{AB} := \mathrm{tr}\{\overline{\psi}_{\dot\alpha A}\overline{\psi}_B^{\dot\alpha}\} \tag{2.5}$$

of conformal dimension $\Delta = 3$, at lowest order in perturbation theory. These operators are in the $SU(4)$ (R-symmetry) irreducible representation **10** (i.e., a symmetric covariant 2-tensor). There is another set of operators of $N = 4$ super Yang-Mills with the same

[1] If we treat ϱ, $\widetilde{\varrho}$ and η as continuous functions of λ and $\tilde\lambda$, the helicity can be alternatively defined as (B.10). These two definitions turn out to be equivalent, as discussed in appendix B.3.

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quantum numbers, at lowest order in perturbation theory. They are cubic in the scalar fields. Let us denote these scalar fields by

$$\phi_{\mathcal{I}}, \qquad \mathcal{I} = 1, \ldots, 6. \tag{2.6}$$

Here $\mathcal{I}$ is an R-symmetry index in the fundamental representation of $so(6) \simeq su(4)$. (For convenience, we present some relevant identities in appendix A.2.)

The second set of operators of conformal dimension $\Delta = 3$ (at 0^{th} order of perturbation theory) and $so(6) \simeq su(4)$ representation **10** can now be written as

$$\mathcal{V}''_{AB} := \Gamma^{\mathcal{I}\mathcal{J}\mathcal{K}}_{AB} \operatorname{tr}\{\phi_{\mathcal{I}}\phi_{\mathcal{J}}\phi_{\mathcal{K}}\}, \tag{2.7}$$

using the $SU(4)$-invariant symbol $\Gamma^{\mathcal{I}\mathcal{J}\mathcal{K}}_{AB}$, defined at the end of appendix A.2. This symbol is anti-symmetric in the $so(6)$ indices $\mathcal{I}\mathcal{J}\mathcal{K}$ and symmetric in the $su(4)$ indices AB, and it connects the representation **10** of $so(6)$ (self-dual 3-tensors) to the representation **10** of $su(4)$.

There is a linear combination of $\mathcal{V}'_{AB}$ and $\mathcal{V}''_{AB}$ that lies in a short supermultiplet. This is the combination

$$\mathcal{V}_{AB} := \mathcal{V}'_{AB} + \frac{1}{4}\mathcal{V}''_{AB}, \tag{2.8}$$

and its conformal dimension $\Delta = 3$ is exact. These operators can be obtained by acting with two supersymmetry transformations on the chiral primary operators $\mathcal{V}_{\mathcal{I}\mathcal{J}} := \operatorname{tr}\{\phi_{\mathcal{I}\mathcal{J}}\}$. (See [11][12] for more details.)

In the next section we will calculate 4-point tree level scattering amplitudes in the presence of the perturbations discussed above. We will include both the 2-fermion and the 3-scalar perturbations in the combination

$$g^2\delta\mathcal{L} = \frac{1}{2}M^{AB}\mathcal{V}_{AB}, \tag{2.9}$$

where g is the Yang-Mills coupling constant and the unperturbed Lagrangian is presented in (B.1).

In [13] it was shown that twistor string theory contains a sector that is described by $N = 4$ conformal supergravity (CSUGRA). Furthermore, tree-level amplitudes in CSUGRA have been calculated in [14] using twistor string theory. The fields of CSUGRA couple to the fields of $N = 4$ Super Yang-Mills (SYM). To linear order, each CSUGRA field couples to an $N = 4$ SYM operator from the short supermultiplet of the chiral primary field $\mathcal{V}_{\mathcal{I}\mathcal{J}}$. For example, CSUGRA contains an $SU(4)$ gauge field that couples to the R-symmetry current of $N = 4$ SYM. CSUGRA also contain scalar fields in the representation $\overline{\mathbf{10}}$ of $SU(4)$, which were denoted by $\overline{E}^{AB}$ in [13]. To linear order, these fields couple to the $N = 4$ SYM operators $\mathcal{V}_{AB}$, and the mass terms that we are considering here can be interpreted as VEVs,

$$M^{AB} \sim \left\langle \overline{E}^{AB} \right\rangle, \tag{2.10}$$

as suggested in [1].

3. Extended MHV amplitudes

In this section we will calculate several scattering amplitudes with a chiral mass term for tree-level planar diagrams. The mass term mainly changes the Feynman diagram rules for the fermions and 3-scalar interaction. We present the fermion propagators and external wavefunctions in figure 1 – figure 3, and the 3-scalar vertex in figure 4. All the relevant Feynman rules are given in appendix B.

When we label the helicity of the amplitude, we use the convention that all external particles are incoming. For example $A(+1,+1,-1,-1)$ represents the amplitude with two incoming helicity $+1$ and two incoming helicity -1 gluons. On the other hand, for convenience, the convention depicted in the figures (and discussed in appendix B) will be that the helicity and momentum are all physical (2 incoming and 2 outgoing particles with their physical momenta and helicities).[2] For the planar diagrams with external particle indices i cyclically attached, all amplitudes include an overall group theory factor $tr[T_1T_2\cdots T_i\cdots T_n]$, which will be suppressed hereafter.

Maximally Helicity Violating (MHV) amplitudes at the tree level are originally defined [15]–[17] as those satisfying the condition $\sum_i(2h_i-2)=-8$ with h_i the helicities of external legs (defined as all incoming). Since the chiral mass term is interpreted as a VEV of a spacetime conformal supergravity field $\overline{E}^{AB}$ (see (2.10)) we can think of the amplitudes with n external legs that contain the mass parameter at order $\mathcal{O}(M^k)$ as coming from diagrams with $(n+k)$ legs, of which k legs correspond to a background CSUGRA field $\overline{E}^{AB}$. The helicity of this field is 0, and therefore mass-deformed SYM diagrams at order $\mathcal{O}(M^k)$ that satisfy $\sum_i(2h_i-2)=-8+2k$ correpond to MHV diagrams in CSUGRA. We can therefore generalize the term "MHV" to *"extended MHV"* to describe those diagrams at order $\mathcal{O}(M^k)$ that satisfy $\sum_i(2h_i-2)=-8+2k$.[3] The holomorphic structure of generalized MHV amplitudes calculated in this section will be discussed in section 5.

We will now present the results of the calculation of various (extended) MHV amplitudes. The Feynman diagrams and the detailed calculation are shown in appendix C for interested readers. We begin with M^{AB}-independent contribution to the MHV amplitudes. These diagrams are the same as those of the undeformed theory, and were calculated in [15][3] with external gluons and in [18][19] with external gluinos. We present them here for completeness, featuring the use of the spinor notation.

[2] This difference in conventions corresponds to replacing $p^\mu\to -p^\mu$, or $\lambda\to i\lambda$ and $\tilde{\lambda}\to i\tilde{\lambda}$ for the outgoing particles. This does not affect our result because we scale $(\lambda_{i1},\lambda_{i2})$ to $(1, Z_i=\lambda_{i2}/\lambda_{i1})$ in the end. However, the form of the momentum conservation condition depends on the convention: $p_1+\cdots+p_4=0$ for "incoming" momenta while $p_1+p_2=p_3+p_4$ for "physical" momenta. The former leads to $\frac{[1,3]}{[1,4]}=-\frac{\langle 2,4\rangle}{\langle 2,3\rangle}$, $\frac{[2,1]}{[2,3]}=-\frac{\langle 4,3\rangle}{\langle 4,1\rangle}$ and so on, while the latter gives $\frac{[1,3]}{[1,4]}=-\frac{\langle 2,4\rangle}{\langle 2,3\rangle}$ and $\frac{[2,1]}{[2,3]}=\frac{\langle 4,3\rangle}{\langle 4,1\rangle}$ (an extra minus sign may arise).

[3] At tree level, however, we only have $\mathcal{O}(M^0)$ and $\mathcal{O}(M)$; the amplitudes at higher orders of M all vanish. This can be understood by (5.7), in which M gives 3 θ_2's but the integrand needs to have exactly 4 θ_2 to yield nonzero result.

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3.1 MHV amplitudes (extended MHV at $\mathcal{O}(M^0)$)

- 4-gluon scattering amplitude:[4]

$$A_{\mathcal{O}(M^0)}(+1,+1,-1,-1) = \frac{ig^2}{2}\frac{\langle 3,4\rangle^4}{\prod_{i=1}^4\langle i,i+1\rangle}. \tag{3.1}$$

- 2-gluon and 2-fermion scattering amplitude:

$$A_{\mathcal{O}(M^0)}(+1/2,+1,-1,-1/2) = ig^2\varrho_4^A\widetilde{\varrho}_{1A}\ \frac{\langle 3,4\rangle^3\langle 1,3\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}. \tag{3.2}$$

- 4-fermion scattering amplitude:

$$\begin{aligned} &A_{\mathcal{O}(M^0)}(+1/2,+1/2,-1/2,-1/2)\\ &= \frac{2ig^2\langle 3,4\rangle^2}{\prod_{i=1}^4\langle i,i+1\rangle}\Big\{\varrho_4^A\widetilde{\varrho}_{1A}\widetilde{\varrho}_{2B}\varrho_3^B\langle 1,3\rangle\langle 2,4\rangle + \varrho_3^A\widetilde{\varrho}_{1A}\widetilde{\varrho}_{2B}\varrho_4^B\langle 2,3\rangle\langle 4,1\rangle\Big\}, \end{aligned} \tag{3.3}$$

- 2-fermion and 2-scalar scattering amplitude:

$$\begin{aligned} &A_{\mathcal{O}(M^0)}(+1/2,0,0,-1/2)\\ &= \frac{2ig^2\langle 3,4\rangle\langle 2,4\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}\left\{\frac{1}{2}\varrho_4^A\widetilde{\varrho}_{1A}\varphi_2^{BC}\varphi_{3BC}\langle 1,2\rangle\langle 3,4\rangle + \varrho_4^B\widetilde{\varrho}_{1A}\varphi_2^{CA}\varphi_{3BC}\langle 2,3\rangle\langle 4,1\rangle\right\}, \end{aligned} \tag{3.4}$$

  where φ_2 and φ_3 are wavefunctions for the external scalars.

3.2 Extended MHV amplitudes at $\mathcal{O}(M)$

- 2-gluon and 2-fermion scattering amplitude:

$$A_{\mathcal{O}(M)}(+1/2,+1,-1,+1/2) = \frac{ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}}{2}\ \frac{\langle 3,1\rangle\langle 3,4\rangle\langle 4,1\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}. \tag{3.5}$$

- 4-fermion scattering amplitude:

$$\begin{aligned} &A_{\mathcal{O}(M)}(+1/2,+1/2,-1/2,+1/2)\\ &= \frac{2ig^2}{\prod_1^4\langle i,i+1\rangle}\Big\{\widetilde{\varrho}_{1A}M^{AB}\widetilde{\varrho}_{2B}\varrho_3^C\widetilde{\varrho}_{4C}\langle 1,2\rangle\langle 2,3\rangle\langle 3,1\rangle\\ &\quad+\widetilde{\varrho}_{1A}\varrho_3^A\widetilde{\varrho}_{2B}M^{BD}\widetilde{\varrho}_{4D}\langle 2,4\rangle\langle 2,3\rangle\langle 3,4\rangle + \widetilde{\varrho}_{2B}\varrho_3^B\widetilde{\varrho}_{1A}M^{AD}\widetilde{\varrho}_{4D}\langle 4,1\rangle\langle 1,3\rangle\langle 3,4\rangle\Big\}. \end{aligned} \tag{3.6}$$

[4]The notations $\langle i,j\rangle$ and $[i,j]$ are short for $\langle\lambda_i,\lambda_j\rangle$ and $[\tilde{\lambda}_i,\tilde{\lambda}_j]$ respectively. We also set $\lambda_{n+1}\equiv\lambda_1$ for n external legs.

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- 2-fermion and 2-scalar scattering amplitude:

$$\begin{aligned} & A_{\mathcal{O}(M)}(+1/2,0,0,+1/2) \\ = & \frac{ig^2}{\prod_{i=1}^4\langle i,i+1\rangle}\Big\{\langle 2,3\rangle\langle 3,4\rangle\langle 4,2\rangle\widetilde{\varrho}_{4C}M^{CB}\varphi_{3BD}\varphi_2^{DA}\widetilde{\varrho}_{1A} \\ & +\langle 1,2\rangle\langle 2,3\rangle\langle 3,1\rangle\widetilde{\varrho}_{4B}\varphi_3^{BD}\varphi_{2DA}M^{AD}\widetilde{\varrho}_{1D} \\ & \frac{1}{2}\left(\langle 1,2\rangle\langle 2,3\rangle\langle 3,4\rangle+\langle 1,2\rangle\langle 1,4\rangle\langle 3,4\rangle\right)\widetilde{\varrho}_{4B}M^{BC}\widetilde{\varrho}_{1C}\varphi_3^{B'D'}\varphi_{2B'D'}\Big\}. \end{aligned} \quad (3.7)$$

4. Chiral B-model mass terms

We now compare the amplitudes calculated in section 3 with an integral over the moduli space of holomorphic curves in twistor space. Let us begin by reviewing some facts about super twistor space [3]. We denote the homogeneous coordinates of the B-model target space $\mathbb{CP}^{3|4}\setminus\mathbb{CP}^{1|4}$ by

$$Z_1=\lambda^1,\quad Z_2=\lambda^2,\quad Z_3=\mu_{\dot{1}},\quad Z_4=\mu_{\dot{2}},\quad \Theta^1,\ldots,\Theta^4.$$

It is convenient to define the two patches

$$U{:=}\{Z^1\neq 0\};\qquad U'{:=}\{Z^2\neq 0\}. \quad (4.1)$$

On the patch U, the set

$$Z{:=}\frac{Z_2}{Z_1},\quad X{:=}\frac{Z_3}{Z_1},\quad Y{:=}\frac{Z_4}{Z_1},\quad \Psi^A{:=}\frac{\Theta^A}{Z_1},$$

is a good coordinate system. On U',

$$Z'{:=}\frac{Z_1}{Z_2}=\frac{1}{Z},\quad X'{:=}\frac{Z_3}{Z_2}=\frac{X}{Z},\quad Y'{:=}\frac{Z_4}{Z_2}=\frac{Y}{Z},\quad \Psi'^A{:=}\frac{\Theta^A}{Z_2}=\frac{1}{Z}\Psi^A, \quad (4.2)$$

is a good coordinate system.

Given a meromorphic function

$$\begin{aligned} \mathcal{A}(X,Y,Z,\Psi^1,\ldots,\Psi^4) = & A+\Psi^A\chi_A+\tfrac{1}{2}\Psi^A\Psi^B\phi_{AB} \\ & +\tfrac{1}{6}\epsilon_{ABCD}\Psi^A\Psi^B\Psi^C\tilde{\chi}^D+\tfrac{1}{24}\epsilon_{ABCD}\Psi^A\Psi^B\Psi^C\Psi^D G, \end{aligned} \quad (4.3)$$

where $A,\chi_A,\phi_{AB},\tilde{\chi}^D,G$ are holomorphic functions of X,Y,Z with possible poles at $Z=0$ and $Z=\infty$, we can construct an on-shell wave-function of the $N=4$ fermion fields by (see appendix of [3]),

$$\begin{aligned} \psi^{\alpha A}(x) &= \frac{1}{2\pi i}\oint_C \lambda^\alpha\tilde{\chi}^A(x_{1\dot{1}}+x_{2\dot{1}}z,x_{1\dot{2}}+x_{2\dot{2}}z,z)dz,\qquad \lambda^1\equiv 1,\quad \lambda^2\equiv z, \\ \overline{\psi}^{\dot{\alpha}}_A(x) &= \frac{1}{2\pi i}\oint_C \frac{\partial}{\partial x_{1\dot{\alpha}}}\chi_A(x_{1\dot{1}}+x_{2\dot{1}}z,x_{1\dot{2}}+x_{2\dot{2}}z,z)dz. \end{aligned} \quad (4.4)$$

The contour integrals are performed on, say, a circle around the origin. There are also similar expressions for the bosons A,ϕ_{AB},G.

We have a lot of freedom in choosing the holomorphic functions $\tilde{\chi}^A$ and χ_A, and it is only their singular behavior at $z = 0$ and $z = \infty$ that is important, as we will now review. We can deform the path C of the contour integrals (4.4) to a small loop around the origin $z = 0$. This shows that these integrals are only sensitive to the singular behavior of $\tilde{\chi}^A$ and χ_A at $Z = 0$. Adding to $\tilde{\chi}^A$ or χ_A a holomorphic function of X, Y, Z that is nonsingular for all $Z \neq \infty$ will not affect the physical wave-functions. Similarly, we can perform the integrals (4.4) in the coordinate system X', Y', Z'. In these coordinates we set the superfield $\mathcal{A}'$ to

$$\mathcal{A}'(X', Y', Z', \Psi'^1, \ldots, \Psi'^4) = \mathcal{A}(X, Y, Z, \Psi^1, \ldots, \Psi^4).$$

The components of this field are

$$\mathcal{A}' = A' + \Psi'^A \chi'_A + \tfrac{1}{2}\Psi'^A \Psi'^B \phi'_{AB} + \tfrac{1}{6}\epsilon_{ABCD}\Psi'^A \Psi'^B \Psi'^C \tilde{\chi}'^D + \tfrac{1}{24}\epsilon_{ABCD}\Psi'^A \Psi'^B \Psi'^C \Psi'^D G',$$

Thus, the transformation rules for the fermionic components are

$$\tilde{\chi}'^A(X', Y', Z') = \frac{1}{Z'^3}\tilde{\chi}^A(\frac{X'}{Z'}, \frac{Y'}{Z'}, \frac{1}{Z'}), \qquad \chi'_A(X', Y', Z') = \frac{1}{Z'}\chi_A(\frac{X'}{Z'}, \frac{Y'}{Z'}, \frac{1}{Z'}).$$

In these variables, the contour integrals (4.4) can be written as

$$\begin{aligned}
\psi^{\alpha A}(x) &= -\frac{1}{2\pi i}\oint_C \lambda'^\alpha \frac{1}{z'^3}\tilde{\chi}'^A(x_{1\dot{1}}z' + x_{2\dot{1}}, x_{1\dot{2}}z' + x_{2\dot{2}}, z')dz', \qquad \lambda'^1 \equiv z', \quad \lambda'^2 \equiv 1, \\
\overline{\psi}^{\dot{\alpha}}_A(x) &= -\frac{1}{2\pi i}\oint_C \frac{1}{z'}\frac{\partial}{\partial x_{1\dot{\alpha}}}\chi'_A(x_{1\dot{1}}z' + x_{2\dot{1}}, x_{1\dot{2}}z' + x_{2\dot{2}}, z')dz' \\
&= -\frac{1}{2\pi i}\oint_C \frac{\partial}{\partial x_{2\dot{\alpha}}}\chi'_A(x_{1\dot{1}}z' + x_{2\dot{1}}, x_{1\dot{2}}z' + x_{2\dot{2}}, z')dz'.
\end{aligned} \tag{4.5}$$

We require that the fields $\tilde{\chi}'^A(X', Y', Z')$ and $\chi'_A(X', Y', Z')$ be holomorphic in X', Y', Z' for all finite X', Y' and all nonzero and finite Z'. But we allow singularities at $Z' = 0$. In fact, similarly to the case of (4.4), the contour integrals are only sensitive to the singular behavior of the fields at $Z' = 0$. Thus, adding to $\mathcal{A}'$ a holomorphic function of $X', Y', Z', \Psi'^1, \ldots, \Psi'^4$ that is nonsingular at $Z' = 0$ will not affect the physical wavefunctions.

To summarize, there is a freedom in the choice of $\mathcal{A}$,

$$\mathcal{A}(X, Y, Z, \Psi) \sim \mathcal{A}(X, Y, Z, \Psi) + \mathcal{A}_0(X, Y, Z, \Psi) + \mathcal{A}_\infty(X, Y, Z, \Psi), \tag{4.6}$$

where $\mathcal{A}_0$ is an arbitrary meromorphic wavefunction that is holomorphic at $Z \neq \infty$ (including $Z = 0$), and $\mathcal{A}_\infty$ is an arbitrary meromorphic wavefunction that is holomorphic at $Z \neq 0$ (including $Z = \infty$). To check the holomorphicity requirement for $\mathcal{A}_\infty$ one has to know what the good coordinates near $Z = \infty$ are. In the undeformed case, these are given by (4.2).

In the next subsection we will reverse this logic and find a deformation of the complex structure that corresponds to a chiral mass term. The idea is as follows. First we find a solution to the Dirac equation (2.1) in a form that augments (4.4). It will be given in

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terms of meromorphic functions on twistor space that we will denote again by $\tilde{\chi}^A$ and χ_A. Then, we define a superfield similarly to (4.3), and we look for an equivalence in the form (4.6). Since (4.4) will be augmented, invariance of the physical wavefunctions $\psi^{\alpha A}$ and $\overline{\psi}^{\dot{\alpha}}_A$ under (4.6) will require a different definition of "holomorphic at $Z=\infty$." This will yield an augmentation of the transition functions (4.2), which will give us the desired deformation of the complex structure. Let's move on to the details!

4.1 Super-complex structure deformation

As explained in [1], a chiral mass term can be incorporated into the B-model twistor string theory as a certain supercomplex structure deformation. General deformations of the complex structure of weighted projective superspaces (and other holomorphic vector bundles) were studied in [20]-[23].

A supercomplex structure deformation can be described by changing the transition functions (4.2). The new transition functions that we need turned out to be

$$Z'=\frac{1}{Z},\quad X'=\frac{X}{Z},\quad Y'=\frac{Y}{Z},\quad \Psi'^A=\frac{1}{Z}\Psi^A+\frac{1}{6Z^2}M^{AB}\epsilon_{BCDE}\Psi^C\Psi^D\Psi^E. \tag{4.7}$$

Let us recall how this deformation was derived in [1]. We start with the free-field Dirac equations (2.1). The generic solution was given in (2.3). There, $\widetilde{\varrho}_A$ and ϱ^A are both functions of λ and $\tilde{\lambda}$, but we can Fourier transform them with respect to $\tilde{\lambda}$ to obtain functions of twistor space that we denote by χ_A and $\tilde{\chi}^A$. We can then write the solution to the Dirac equation (2.1) as

$$\begin{aligned}\psi^A_\alpha(x)&=\frac{1}{2\pi i}\oint_C\left[\lambda_\alpha\tilde{\chi}^A(z,x_{1\dot{1}}+x_{2\dot{1}}z,x_{1\dot{2}}+x_{2\dot{2}}z)+M^{AB}\eta_\alpha\chi_B(z,x_{1\dot{1}}+x_{2\dot{1}}z,x_{1\dot{2}}+x_{2\dot{2}}z)\right]dz,\\ \overline{\psi}^{\dot{\alpha}}_A(x)&=\frac{1}{2\pi i}\oint_C\frac{\partial}{\partial x_{1\dot{\alpha}}}\chi_A(z,x_{1\dot{1}}+x_{2\dot{1}}z,x_{1\dot{2}}+x_{2\dot{2}}z)dz,\end{aligned} \tag{4.8}$$

where

$$(\lambda^1,\lambda^2)\equiv(1,z),\qquad(\eta_1,\eta_2)\equiv(1,0).$$

[Equations (4.8) can be compared to (4.4) in the massless case.] We can collect χ_A and $\tilde{\chi}^A$ in a superfield as in (4.3). Let us see what would be the analog of the equivalence relation (4.6). Obviously, $\overline{\psi}^{\dot{\alpha}}_A$ and $\psi_{\alpha A}$ in (4.8) do not change if we add an arbitrary holomorphic function at $z\neq\infty$ to either $\tilde{\chi}^A$ or χ_A or both. Thus, the equivalence $\mathcal{A}\sim\mathcal{A}+\mathcal{A}_0$ is the same as in the massless case (4.6).

Things change, however, for $\mathcal{A}_\infty$ in (4.6). The coordinates X',Y',Z' from (4.2) are no longer good coordinates near $Z=\infty$. If they were, we could define

$$\chi'_A(Z,X,Y)=\frac{1}{Z}\chi_A(\frac{1}{Z},\frac{X}{Z},\frac{Y}{Z}),\qquad\tilde{\chi}''^A(Z,X,Y)=\frac{1}{Z^3}\tilde{\chi}^A(\frac{1}{Z},\frac{X}{Z},\frac{Y}{Z}), \tag{4.9}$$

(we use $\tilde{\chi}''^A$ because it is only a temporary expression and we will modify it below) to change, and we could write the integrals in (4.8) as follows. We could substitute (4.9) in

the second equation of (4.8) to get

$$\begin{aligned}\overline{\psi}^{\dot\alpha}_A(x) &= \frac{1}{2\pi i}\oint_C \frac{1}{z}\frac{\partial}{\partial x_{1\dot\alpha}}\chi_A(\frac{1}{z},\frac{x_{1\dot 1}}{z}+x_{2\dot 1},\frac{x_{1\dot 2}}{z}+x_{2\dot 2})dz \\ &= -\frac{1}{2\pi i}\oint_C \frac{\partial}{\partial x_{2\dot\alpha}}\chi_A(z',x_{1\dot 1}z'+x_{2\dot 1},x_{1\dot 2}z'+x_{2\dot 2})dz'\end{aligned} \tag{4.10}$$

This last integrand is regular near $z'=0$ and therefore vanishes if χ_A is holomorphic near $z'=0$, as in the massless case. However, there would be a problem with the integrand for $\overline{\psi}^{\dot\alpha}_A$. Substituting (4.9) in the first equation of (4.8), we would get

$$\begin{aligned}\psi^A_\alpha(x) &= \frac{1}{2\pi i}\oint_C\Big[\lambda_\alpha\frac{1}{z^3}\tilde\chi''^A\Big(\frac{1}{z},\frac{x_{1\dot 1}}{z}+x_{2\dot 1},\frac{x_{1\dot 2}}{z}+x_{2\dot 2}\Big)+ \\ &\quad+\frac{1}{z}M^{AB}\eta_\alpha\chi'_B\Big(\frac{1}{z},\frac{x_{1\dot 1}}{z}+x_{2\dot 1},\frac{x_{1\dot 2}}{z}+x_{2\dot 2}\Big)\Big]dz \\ &= -\frac{1}{2\pi i}\oint_C\Big[\lambda_\alpha z'\tilde\chi''^A(z',x_{1\dot 1}z'+x_{2\dot 1},x_{1\dot 2}z'+x_{2\dot 2})+ \\ &\quad+\frac{1}{z'}M^{AB}\eta_\alpha\chi'_B(z',x_{1\dot 1}z'+x_{2\dot 1},x_{1\dot 2}z'+x_{2\dot 2})\Big]dz'\end{aligned} \tag{4.11}$$

If $\tilde\chi''^A$ is regular at $z'=\infty$, the first term in the integrand is regular, because for $\alpha=2$ we have $\lambda_\alpha=1$ and for $\alpha=1$ we have $\lambda_\alpha=-z=-1/z'$. However, the second term may have a pole for $\alpha=1$ because $\eta_1=1$. The integrand of (4.11) therefore does not necessarily vanish. This shows that (4.9) is incompatible with (4.6). We can fix this problem by a slight modification of (4.9). We define instead,

$$\begin{aligned}\chi'_A(Z,X,Y) &= \frac{1}{Z}\chi_A\left(\frac{1}{Z},\frac{X}{Z},\frac{Y}{Z}\right), \\ \tilde\chi'^A(Z,X,Y) &= \frac{1}{Z^3}\tilde\chi^A\left(\frac{1}{Z},\frac{X}{Z},\frac{Y}{Z}\right)-\frac{1}{Z^2}M^{AB}\chi_A\left(\frac{1}{Z},\frac{X}{Z},\frac{Y}{Z}\right).\end{aligned} \tag{4.12}$$

This can be inverted to

$$\begin{aligned}\chi_A(Z,X,Y) &= \frac{1}{Z}\chi'_A\left(\frac{1}{Z},\frac{X}{Z},\frac{Y}{Z}\right), \\ \tilde\chi^A(Z,X,Y) &= \frac{1}{Z^3}\tilde\chi'^A\left(\frac{1}{Z},\frac{X}{Z},\frac{Y}{Z}\right)+\frac{1}{Z^2}M^{AB}\chi'_A\left(\frac{1}{Z},\frac{X}{Z},\frac{Y}{Z}\right).\end{aligned} \tag{4.13}$$

Then

$$\begin{aligned}\psi^A_\alpha(x) &= \frac{1}{2\pi i}\oint_C\big[\lambda_\alpha\frac{1}{z^3}\tilde\chi'^A(\frac{1}{z},\frac{x_{1\dot 1}}{z}+x_{2\dot 1},\frac{x_{1\dot 2}}{z}+x_{2\dot 2}) \\ &\quad+\frac{1}{z}M^{AB}(\eta_\alpha+\frac{1}{z}\lambda_\alpha)\chi'_B(\frac{1}{z},\frac{x_{1\dot 1}}{z}+x_{2\dot 1},\frac{x_{1\dot 2}}{z}+x_{2\dot 2})\big]dz \\ &= -\frac{1}{2\pi i}\oint_C\big[\lambda_\alpha z'\tilde\chi'^A(z',x_{1\dot 1}z'+x_{2\dot 1},x_{1\dot 2}z'+x_{2\dot 2}) \\ &\quad+M^{AB}(\frac{1}{z'}\eta_\alpha+\lambda_\alpha)\chi_B(z',x_{1\dot 1}z'+x_{2\dot 1},x_{1\dot 2}z'+x_{2\dot 2})\big]dz'\end{aligned} \tag{4.14}$$

Now the integrand is regular at $z' = \infty$ because

$$\frac{1}{z'}\eta_1 + \lambda_1 \overset{z'\to\infty}{\longrightarrow} 0, \qquad \frac{1}{z'}\eta_2 + \lambda_2 \overset{z'\to\infty}{\longrightarrow} 1.$$

Thus, the field redefinition (4.12) is compatible with the equivalence relation (4.6). These redefinitions (4.12) are the Ψ and $\Psi\Psi\Psi$ components of the superfield expression

$$\mathcal{A}'(X', Y', Z', \Psi'^A) = \mathcal{A}(X, Y, Z, \Psi^A),$$

where the coordinates X', Y', Z', Ψ'^A are defined in (4.7).

4.2 A note on the anti-chiral mass term

One might wonder whether we could derive a similar modification of the complex structure of super twistor space for the anti-chiral mass term deformation $M_{AB}\psi^A_\alpha\psi^{\alpha B}$. In this case, instead of the Dirac equations (2.1) we get

$$p_{\alpha\dot\alpha}\overline{\psi}^{\dot\alpha}_A = M_{AB}\psi^B_\alpha, \qquad p_{\alpha\dot\alpha}\psi^{\alpha A} = 0.$$

Instead of (4.8), the solution is now given by

$$\begin{aligned}
\psi^A_\alpha(x) &= \frac{1}{2\pi i}\oint_C \lambda_\alpha \tilde\chi^A(z, x_{1\dot 1} + x_{2\dot 1}z, x_{1\dot 2} + x_{2\dot 2}z)\big]dz,\\
\overline{\psi}^{\dot\alpha}_A(x) &= \frac{1}{2\pi i}\oint_C \Big[\frac{\partial}{\partial x_{1\dot\alpha}}\chi_A(z, x_{1\dot 1} + x_{2\dot 1}z, x_{1\dot 2} + x_{2\dot 2}z) + \\
&\quad + M_{AB}\hat\eta_{\dot\alpha}\tilde\chi^B(z, x_{1\dot 1} + x_{2\dot 1}z, x_{1\dot 2} + x_{2\dot 2}z)\Big]dz.
\end{aligned} \tag{4.15}$$

Here $\hat\eta_{\dot\alpha}$ has to satisfy

$$\tilde\lambda^{\dot\alpha}\hat\eta_{\dot\alpha} = 1.$$

In twistor space, however, we identify $\tilde\lambda^{\dot\alpha}$ with a differential operator

$$\tilde\lambda^{\dot\alpha} \leftrightarrow i\frac{\partial}{\partial\mu_{\dot\alpha}}.$$

We can therefore set $\hat\eta_{\dot\alpha}$ to be the following integral operator

$$\hat\eta_{\dot 1} f(z, \mu_{\dot 1}, \mu_{\dot 2}) := 0, \qquad \hat\eta_{\dot 2} f(z, \mu_{\dot 1}, \mu_{\dot 2}) := -i\int_0^{\mu_{\dot 2}} f(z, \mu_{\dot 1}, s)ds, \tag{4.16}$$

where f is an arbitrary holomorphic function. Now the field redefinition (4.9) is good enough:

$$\begin{aligned}
\overline{\psi}^{\dot\alpha}_A(x) &= \frac{1}{2\pi i}\oint_C \big[\frac{\partial}{\partial x_{1\dot\alpha}}\chi_A(z, x_{1\dot 1} + x_{2\dot 1}z, x_{1\dot 2} + x_{2\dot 2}z) + \\
&\qquad + M_{AB}\hat\eta_{\dot\alpha}\tilde\chi^B(z, x_{1\dot 1} + x_{2\dot 1}z, x_{1\dot 2} + x_{2\dot 2}z)\big]dz\\
&= \frac{1}{2\pi i}\oint_C \big[\frac{1}{z}\frac{\partial}{\partial x_{1\dot\alpha}}\chi'_A(\frac{1}{z}, \frac{1}{z}x_{1\dot 1} + x_{2\dot 1}, \frac{1}{z}x_{1\dot 2} + x_{2\dot 2}) \\
&\qquad + \frac{1}{z^3}M_{AB}\hat\eta_{\dot\alpha}\tilde\chi''^B(\frac{1}{z}, \frac{1}{z}x_{1\dot 1} + x_{2\dot 1}, \frac{1}{z}x_{1\dot 2} + x_{2\dot 2})\big]dz\\
&= -\frac{1}{2\pi i}\oint_C \big[z'\frac{\partial}{\partial x_{1\dot\alpha}}\chi'_A(z', x_{1\dot 1}z' + x_{2\dot 1}, x_{1\dot 2}z' + x_{2\dot 2}) \\
&\qquad + z'^3 M_{AB}\hat\eta_{\dot\alpha}\tilde\chi''^B(z', x_{1\dot 1}z' + x_{2\dot 1}, x_{1\dot 2}z' + x_{2\dot 2})\big]\frac{dz'}{z'^2}
\end{aligned}$$

Using (4.16) we see that for $\dot{\alpha} = \dot{1}$ the integrand is regular at $z' = 0$. For $\dot{\alpha} = \dot{2}$ we get

$$\overline{\psi}^{\dot{2}}_A(x) = -\frac{1}{2\pi i}\oint_C \left[z'^2 \frac{\partial}{\partial x_{2\dot{2}}} \chi'_A(z', x_{1\dot{1}} z' + x_{2\dot{1}}, x_{1\dot{2}} z' + x_{2\dot{2}}) \right.$$
$$\left. + z'^3 M_{AB} \int_0^{x_{1\dot{2}} z' + x_{2\dot{2}}} \tilde{\chi}'^B(z', x_{1\dot{1}} z' + x_{2\dot{1}}, s') \frac{ds'}{z'} \right] \frac{dz'}{z'^2},$$

where we used the definition (4.16) for $\dot{\alpha} = \dot{2}$, and we changed variables from s to $s' = z's$ in the integral that defines $\hat{\eta}_{\dot{2}}$. We now see that the integrand in (4.17) is regular at $z' = \infty$. Therefore, no deformation of complex structure is needed! However, it was suggested in [1], that the anti-chiral mass parameter M_{AB} enters in a phase of D1-instanton terms. The argument was based on a proposed indentification of M_{AB} with a VEV of one of the conformal supergravity fields discussed in [13]. We will not explore this further in the present paper.

4.3 Deformed holomorphic curve

In the undeformed twistor space, a curve of degree $d = 1$ in $\mathbb{CP}^{3|4}$ is given by (see equation (4.46) of [3]) a set of linear equations:

$$X = -x_{1\dot{1}} - x_{2\dot{1}} Z, \qquad Y = -x_{1\dot{2}} - x_{2\dot{2}} Z, \qquad \Psi^A = -\theta_1^A - \theta_2^A Z, \tag{4.17}$$

where $x_{\alpha\dot{\alpha}}$ and θ^A_α are moduli. On the patch U' [defined in (4.1)] we can write (4.17) as

$$X' = -x_{1\dot{1}} Z' - x_{2\dot{1}}, \qquad Y' = -x_{1\dot{2}} Z' - x_{2\dot{2}}, \qquad \Psi^A = -\theta_1^A Z' - \theta_2^A. \tag{4.18}$$

After the deformation (4.7), equations (4.18) no longer hold. Instead, equations (4.17) imply

$$\begin{aligned}
\Psi'^A &= \frac{1}{Z}\Psi^A + \frac{1}{6Z^2} M^{AB} \epsilon_{BCDE} \Psi^C \Psi^D \Psi^E \\
&= -\theta_1^A Z' - \theta_2^A - \frac{1}{6Z'} M^{AB} \epsilon_{BCDE} (\theta_1^C Z' + \theta_2^C)(\theta_1^D Z' + \theta_2^D)(\theta_1^E Z' + \theta_2^E) \\
&= -\frac{1}{6} M^{AB} \epsilon_{BCDE} \theta_1^C \theta_1^D \theta_1^E Z'^2 - \left(\theta_1^A + \frac{1}{2} M^{AB} \epsilon_{BCDE} \theta_1^C \theta_1^D \theta_2^E \right) Z' \\
&\quad - \left(\theta_2^A + \frac{1}{2} M^{AB} \epsilon_{BCDE} \theta_1^C \theta_2^D \theta_2^E \right) - \frac{1}{2Z'} M^{AB} \epsilon_{BCDE} \theta_2^C \theta_2^D \theta_2^E.
\end{aligned}$$

Unless the last term is zero, this is not an acceptable holomorphic curve. We can cancel the last term by slightly modifying equations (4.17) in the patch U as follows. We set

$$\Psi^A = -\theta_1^A - \theta_2^A Z + \frac{1}{6} M^{AB} \epsilon_{BCDE} \theta_2^C \theta_2^D \theta_2^E Z^2. \tag{4.19}$$

Then

$$\begin{aligned}
\Psi'^A &= \frac{1}{Z}\Psi^A + \frac{1}{6Z^2} M^{AB} \epsilon_{BCDE} \Psi^C \Psi^D \Psi^E \\
&= -\frac{1}{6} M^{AB} \epsilon_{BCDE} \theta_1^C \theta_1^D \theta_1^E Z'^2 - \left(\theta_1^A + \frac{1}{2} M^{AB} \epsilon_{BCDE} \theta_1^C \theta_1^D \theta_2^E \right) Z'
\end{aligned}$$

$$-\left(\theta_2^A+\frac{1}{2}M^{AB}\epsilon_{BCDE}\theta_1^C\theta_2^D\theta_2^E+\frac{1}{2}M^{AB}\epsilon_{BCDE}M^{EF}\epsilon_{FGHI}\theta_1^C\theta_1^D\theta_2^G\theta_2^H\theta_2^I\right)$$

The poles in Z' vanish — the terms proportional to $1/Z'^2$ vanishes because there are too many θ_2's, and the terms proportional to $1/Z'$ vanish because the symmetric mass parameter M^{AB} is coupled with the antisymmetric $\epsilon-$tensor. In the next section we will express the scattering amplitudes calculated in section 3 as an integral over supertwistor space with support on the deformed holomorphic curve that we just found:

$$\begin{aligned} X&=-x_{1\dot{1}}-x_{2\dot{1}}Z, \qquad Y=-x_{1\dot{2}}-x_{2\dot{2}}Z,\\ \Psi^A&=-\theta_1^A-\theta_2^AZ+\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^EZ^2. \end{aligned} \tag{4.20}$$

5. Twistor amplitudes with mass terms

As explained in [3], a supersymmetric Yang-Mills amplitude $A(f_i)$ can be obtained from a twistor scattering amplitude $\tilde{A}(\lambda_i^\alpha,\mu_i^{\dot\alpha},\Psi_i^A)$ by multiplying by the appropriate external wavefunctions $f_i(\lambda_i^\alpha,\mu_i^{\dot\alpha},\Psi_i^A)$ and integrating out all the supertwistor coordinates. In particular, the tree-level MHV amplitudes for 4 external particles (without the mass deformation) can be written as

$$\begin{aligned} \tilde{A}(\lambda_i^\alpha,\mu_{i\dot\alpha},\Psi_i^A)&=2ig^2\int d^4x\,d^8\theta_a^A\prod_{i=1}^4\delta^2(\mu_{i\dot\alpha}+x_{\alpha\dot\alpha}\lambda_i^\alpha)\delta^4(\Psi_i^A+\theta_\alpha^A\lambda_i^\alpha)\frac{1}{\prod_{i=1}^4\langle i,i+1\rangle},\\ A(f_i)&=\prod_{i=1}^4\int d^2\lambda_i^\alpha d^2\mu_{i\dot\alpha}d^4\Psi_i^Af_i(\lambda_i^\alpha,\mu_{i\dot\alpha},\Psi_i^A)\tilde{A}(\lambda_i^\alpha,\mu_i^{\dot\alpha},\Psi_i^A). \end{aligned} \tag{5.1}$$

The δ-functions in the integral imply that the twistor amplitude vanishes unless the external points $(\lambda_i^\alpha,\mu_{i\dot\alpha},\Psi_i^A)$ all lie in the same holomorphic curve described by

$$\mu_{i\dot\alpha}+x_{\alpha\dot\alpha}\lambda_i^\alpha=0, \qquad \Psi_i^A+\theta_\alpha^A\lambda_i^\alpha=0, \tag{5.2}$$

for some suitable $(x_{\alpha\dot\alpha},\theta_\alpha^A)$.

In the presence of our mass term the complex structure is deformed and, according to (4.20), the holomorphic curve is changed to

$$\mu_{i\dot\alpha}+x_{\alpha\dot\alpha}\lambda_i^\alpha=0, \qquad \Psi_i^A+\theta_1^A+\theta_2^AZ_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^EZ_i^2=0. \tag{5.3}$$

We claim that (5.1) is modified to

$$\begin{aligned} \tilde{A}(\lambda_i'^\alpha,\mu'_{i\dot\alpha},\Psi_i^A)&=2ig^2\int d^4x\,d^8\theta_a^A\frac{1}{\prod_{i=1}^4\langle\lambda'_i,\tilde\lambda'_{i+1}\rangle}\\ &\quad\times\prod_{i=1}^4\delta^2\left(\mu'_{i\dot\alpha}+x_{\alpha\dot\alpha}\lambda_i'^\alpha\right)\delta^4\left(\Psi_i^A+\theta_1^A+\theta_2^AZ'_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^EZ_i'^2\right),\\ A(f_i)&=\prod_{i=1}^4\int d^2\lambda_i'^\alpha d^2\mu'_{i\dot\alpha}d^4\Psi_i^Af_i(\lambda_i'^\alpha,\mu'_{i\dot\alpha},\Psi_i^A)\tilde{A}(\lambda_i'^\alpha,\mu'_{i\dot\alpha},\Psi_i^A) \end{aligned} \tag{5.4}$$

for all "extended MHV" amplitudes (defined in section 3) of 4 external particles.

In particular, if the external function is in a momentum state, f_i is of the form of a Fourier transform of a δ-function peaked at some fixed momentum; i.e.,

$$f_i^{(p_i=\lambda_i\tilde{\lambda}_i)}(\lambda_i'^{\alpha}, \mu'_{i\dot{\alpha}}, \Psi_i^A) = f_i(\Psi_i^A)\frac{1}{(2\pi)^4}\int d^2\tilde{\lambda}_i'^{\dot{\alpha}}\ \delta^2(\lambda_i^{\alpha} - \lambda_i'^{\alpha})\delta^2(\tilde{\lambda}_i^{\dot{\alpha}} - \tilde{\lambda}_i'^{\dot{\alpha}})\exp(i\tilde{\lambda}_i'^{\dot{\alpha}}\mu'_{i\dot{\alpha}}), \tag{5.5}$$

where $p_i^{\alpha\dot{\alpha}} = \lambda_i^{\alpha}\tilde{\lambda}_i^{\dot{\alpha}}$ are the momenta for external particles. The integral over bosonic coordinates in (5.4) yields

$$\begin{aligned}
&\frac{1}{(2\pi)^4}\int d^2\lambda_i'^{\alpha}\, d^2\mu'_{i\dot{\alpha}}\int d^4x\, g(\lambda_i'^{\alpha})\,\delta^2(\mu'_{i\dot{\alpha}} + x_{\alpha\dot{\alpha}}\lambda_i'^{\alpha}) \\
&\qquad\times\int d^2\tilde{\lambda}_i'^{\dot{\alpha}}\,\delta^2(\lambda_i^{\alpha} - \lambda_i'^{\alpha})\delta^2(\tilde{\lambda}_i^{\dot{\alpha}} - \tilde{\lambda}_i'^{\dot{\alpha}})\exp(i\tilde{\lambda}_i'^{\dot{\alpha}}\mu'_{i\dot{\alpha}}) \\
&= \frac{1}{(2\pi)^4}\int d^4x\exp(-i\sum_i x_{\alpha\dot{\alpha}}\lambda_i^{\alpha}\tilde{\lambda}_i^{\dot{\alpha}})\, g(\lambda_i^{\alpha}) = \delta^4(\sum_i p_i)\, g(\lambda_i^{\alpha}),
\end{aligned} \tag{5.6}$$

where $g(\lambda_i'^{\alpha})$ denotes the λ'-dependence of $\tilde{A}(\lambda_i'^{\alpha}, \mu'_{i\dot{\alpha}}, \Psi_i^A)$ apart from $\delta^2(\mu'_{i\dot{\alpha}} + x_{\alpha\dot{\alpha}}\lambda_i'^{\alpha})$ and the result gives rise the momentum conservation factor. Consequently, (5.4) reduces to

$$\begin{aligned}
A(f_i) = 2ig^2\delta^4\left(\sum_i p_i\right)\int\prod_i^4 d^4\Psi_i^A \\
\int d^8\theta_a^A\prod_{i=1}^4\delta^4\left(\Psi_i^A + \theta_1^A + \theta_2^A Z_i - \frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^E Z_i^2\right) \\
\times\frac{1}{\prod_{i=1}^4\langle i, i+1\rangle} f_1(\Psi_1^A)f_2(\Psi_2^A)f_3(\Psi_3^A)f_4(\Psi_4^A),
\end{aligned} \tag{5.7}$$

where the external function $f_i(\Psi_i^A)$ is given by A, $\Psi^A\chi_A$, $\frac{1}{2}\Psi^A\Psi^B\phi_{AB}$, $\frac{1}{6}\epsilon_{ABCD}\Psi^A\Psi^B\Psi^C\tilde{\chi}^D$ and $\frac{1}{24}\epsilon_{ABCD}\Psi^A\Psi^B\Psi^C\Psi^D G$ for the external particle with spin $+1$, $+1/2$, 0, $-1/2$ and -1 respectively.

In the following, we compute various amplitudes in momentum space by (5.7) (ignore the momentum conservation factor) and compare them with the results we have calculated by Feynman rules in section 3, with the identification: $A = G = \frac{1}{\sqrt{2}}$, $\chi = \widetilde{\varrho}$, $\tilde{\chi} = \varrho$ and $\phi = \varphi$. In all cases, both results agree with each other and we thus verify the claim in (5.4). Finally, in section 5.2.4, we study a simple but interesting case — the 3-scalar interaction.

5.1 MHV amplitudes (extended MHV at $\mathcal{O}(M^0)$)

The Grassmanian integral over $d^8\theta_a^A$ in (5.7) gives a nonzero result only if the integrand has exactly 8 Grassman θ's. In the cases of MHV amplitudes, the external functions $f_i(\Psi_i^A)$ altogether have 8 fermionic coordinates Ψ's, each of which gives 1 or 3 θ's by the δ-function. The mass term in (5.3) is of the order $\mathcal{O}(\theta^3)$ and thus has too many θ's to contribute in (5.7) for MHV amplitudes. As a result, the mass deformation does not affect the MHV amplitudes and the amplitudes are the same as if no mass deformation. In the following, we compute various MHV amplitudes by (5.7) and compare them with the results in section 3.1.

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5.1.1 $A_{\mathcal{O}(M^0)}(+1,+1,-1,-1)$

The external functions for this case are A_1, A_2, $\frac{1}{24}\epsilon_{ABCD}\Psi^A\Psi^B\Psi^C\Psi^D G_3$ and $\frac{1}{24}\epsilon_{ABCD}\times\Psi^A\Psi^B\Psi^C\Psi^D G_4$. The integral (5.7) gives the amplitude

$$\begin{aligned}
&2ig^2\int\prod_i^4 d^4\Psi_i^A\int d^8\theta_a^A\prod_{i=1}^4\delta^4\left(\Psi_i^A+\theta_1^A+\theta_2^A Z_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^E Z_i^2\right)\\
&\quad\times\frac{1}{\prod_{i=1}^4\langle i,i+1\rangle}(A_1)(A_2)\left(\frac{1}{24}\epsilon_{ABCD}\Psi_3^A\Psi_3^B\Psi_3^C\Psi_3^D G_3\right)\\
&\quad\times\left(\frac{1}{24}\epsilon_{A'B'C'D'}\Psi_3^{A'}\Psi_3^{B'}\Psi_3^{C'}\Psi_3^{D'}G_4\right)\\
&=2ig^2\frac{A_1A_2G_3G_4}{\prod_{i=1}^4\langle i,i+1\rangle}(Z_3-Z_4)^4=2ig^2\frac{A_1A_2G_3G_4}{\prod_{i=1}^4\langle i,i+1\rangle}\langle 3,4\rangle^4,
\end{aligned}\tag{5.8}$$

which agrees with (3.1).

5.1.2 $A_{\mathcal{O}(M^0)}(+1/2,+1,-1,-1/2)$

The external wavefunctions are $\Psi^A\chi_A$, A_2, $\frac{1}{24}\epsilon_{ABCD}\Psi^A\Psi^B\Psi^C\Psi^D G_3$ and $\frac{1}{6}\epsilon_{ABCD}\Psi^A\Psi^B\times\Psi^C\tilde{\chi}^D$. The amplitude is

$$\begin{aligned}
&2ig^2\int\prod_i^4 d^4\Psi_i^A\int d^8\theta_a^A\prod_{i=1}^4\delta^4\left(\Psi_i^A+\theta_1^A+\theta_2^A Z_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^E Z_i^2\right)\\
&\quad\times\frac{1}{\prod_1^4\langle i,i+1\rangle}\left(\Psi_1^A\chi_{1A}\right)(A_2)\left(\frac{1}{24}\epsilon_{A'B'C'D'}\Psi_3^{A'}\Psi_3^{B'}\Psi_3^{C'}\Psi_3^{D'}G_3\right)\\
&\quad\times\left(\frac{1}{6}\epsilon_{A''B''C''D''}\Psi_4^{A''}\Psi_4^{B''}\Psi_4^{C''}\tilde{\chi}_4^{D''}\right)\\
&=-2ig^2\int d^8\theta_a^A\frac{\chi_{1A}A_2G_3\tilde{\chi}_4^{D''}}{\prod_1^4\langle i,i+1\rangle}7(\theta_1^A+\theta_2^A Z_1+\cdots)\frac{1}{24}\epsilon_{A'B'C'D'}(\theta_1^{A'}+\theta_2^{A'}Z_3+\cdots)\\
&\quad\times(\theta_1^{B'}+\theta_2^{B'}Z_3+\cdots)(\theta_1^{C'}+\theta_2^{C'}Z_3+\cdots)(\theta_1^{D'}+\theta_2^{D'}Z_3+\cdots)\\
&\quad\times\frac{1}{6}\epsilon_{A''B''C''D''}(\theta_1^{A''}+\theta_2^{A''}Z_4+\cdots)(\theta_1^{B''}+\theta_2^{B''}Z_4+\cdots)(\theta_1^{C''}+\theta_2^{C''}Z_4+\cdots)\\
&=2ig^2\frac{(Z_3-Z_1)(Z_4-Z_3)^3}{\prod_1^4\langle i,i+1\rangle}\tilde{\chi}_4^A\chi_{1A}A_2G_3=2ig^2\frac{\langle 1,3\rangle\langle 3,4\rangle^3}{\prod_1^4\langle i,i+1\rangle}\tilde{\chi}_4^A\chi_{1A}A_2G_3,
\end{aligned}\tag{5.9}$$

where $\cdots$ represents the mass-deformed part of the holomorphic curve. The result agrees with (3.2)

5.1.3 $A_{\mathcal{O}(M^0)}(+1/2,+1/2,-1/2,-1/2)$

The external wavefunctions for this amplitude are $\Psi^A\chi_A$, $\Psi^A\chi_A$, $\frac{1}{24}\epsilon_{ABCD}\Psi^A\Psi^B\Psi^C\Psi^D G_3$ and $\frac{1}{24}\epsilon_{ABCD}\Psi^A\Psi^B\Psi^C\Psi^D G_3$. The amplitude is

$$2ig^2\int\prod_i^4 d^4\Psi_i^A\int d^8\theta_a^A\prod_{i=1}^4\delta^4\left(\Psi_i^A+\theta_1^A+\theta_2^A Z_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^E Z_i^2\right)\frac{1}{\prod_1^4\langle i,i+1\rangle}$$

$$
\begin{aligned}
&\times(\Psi_1^A\chi_{1A})(\Psi_2^{A'}\chi_{2A'})\left(\frac{1}{6}\epsilon_{A''B''C''D''}\Psi_3^{A''}\Psi_3^{B''}\Psi_3^{C''}\tilde{\chi}_3^{D''}\right)\\
&\left(\frac{1}{24}\epsilon_{A'''B'''C'''D'''}\Psi^{A'''}\Psi^{B'''}\Psi^{C'''}\Psi^{D'''}G_3\right)\\
&=\frac{2ig^2\langle 3,4\rangle^2}{\prod_1^4\langle i,i+1\rangle}\Big\{\tilde{\chi}_4{}^A\chi_{1A}\chi_{2A'}\tilde{\chi}_3{}^{A'}\Big[(Z_3-Z_1)Z_4^3+(Z_1Z_2-Z_2Z_3-2Z_3^2-2Z_1Z_3)Z_4^2\\
&\qquad+(Z_3^3-Z_1Z_3^2-2Z_2Z_3^2-2Z_1Z_2Z_3)Z_4+Z_1Z_2Z_3^2-Z_2Z_3^3\Big]\\
&+\tilde{\chi}_3{}^A\chi_{1A}\chi_{2A'}\tilde{\chi}_4{}^{A'}\Big[(Z_2-Z_3)Z_4^3+(Z_1Z_3-Z_1Z_2-2Z_3^2-2Z_2Z_3)Z_4^2\\
&\qquad+(-Z_3^3+Z_2Z_3^2-2Z_1Z_3^2+2Z_1Z_2Z_3)Z_4-Z_1Z_2Z_3^3+Z_1Z_3^3\Big]\Big\}\\
&=\frac{2ig^2\langle 3,4\rangle^2}{\prod_1^4\langle i,i+1\rangle}\Big\{\tilde{\chi}_4{}^A\chi_{1A}\chi_{2A'}\tilde{\chi}_3{}^{A'}\langle 1,3\rangle\langle 2,4\rangle+\tilde{\chi}_3{}^A\chi_{1A}\chi_{2A'}\tilde{\chi}_4{}^{A'}\langle 2,3\rangle\langle 4,1\rangle\Big\},
\end{aligned}
\tag{5.10}
$$

which agrees with (3.3).

5.1.4 $A_{\mathcal{O}(M^0)}(+1/2,0,0,-1/2)$

The external wavefunctions are $\Psi_1^A\chi_{1A}$, $\frac{1}{2!}\Psi_2^A\Psi_2^B\phi_{2AB}$, $\frac{1}{2!}\Psi_3^A\Psi_3^B\phi_{3AB}$ and $\frac{1}{3!}\epsilon_{ABCD}\Psi_4^A\times\Psi_4^B\Psi_4^C\tilde{\chi}_4^D$. The amplitude is

$$
\begin{aligned}
&2ig^2\int\prod_i^4 d^4\Psi_i^A\int d^8\theta_a^A\prod_{i=1}^4\delta^4(\Psi_i^A+\theta_1^A+\theta_2^AZ_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^EZ_i^2)\,\frac{1}{\prod_{i=1}^4\langle i,i+1\rangle}\\
&\times(\Psi_1^A\chi_{1A})\left(\frac{1}{2!}\Psi_2^{A'}\Psi_2^{B'}\phi_{2A'B'}\right)\left(\frac{1}{2!}\Psi_3^{A''}\Psi_3^{B''}\phi_{3A''B''}\right)\left(\frac{1}{3!}\epsilon_{A'''B'''C'''D'''}\Psi_4^{A'''}\Psi_4^{B'''}\Psi_4^{C'''}\tilde{\chi}_4^{D'''}\right)\\
&=\frac{2ig^2}{\prod_{i=1}^4\langle i,i+1\rangle}\Big\{\tilde{\chi}_4^{A'}\phi_{3A'B'}\phi_2^{B'A}\chi_{1A}(-Z_3Z_4^3+Z_1Z_3Z_4^2+Z_3^2Z_4^2+2Z_2Z_3Z_4^2+2Z_1Z_2Z_3Z_4\\
&\quad+Z_1Z_2^2Z_4+Z_2^2Z_3Z_4-Z_1Z_2^2Z_3)+\tilde{\chi}_4^{A'}\phi_{2A'B'}\phi_3^{B'A}\chi_{1A}(-Z_2Z_4^3+Z_1Z_2Z_4^2+Z_2^2Z_4^2\\
&\quad+2Z_2Z_3Z_4^2+2Z_1Z_2Z_3Z_4+Z_1Z_3^2Z_4+Z_2Z_3^2Z_4-Z_1Z_2Z_3^2)\\
&\quad+\frac{1}{2}\tilde{\chi}_4^A\chi_{1A}\phi^{3A'B'}\phi_{2A'B'}(-Z_1Z_4^3-Z_2^2Z_3^2+2Z_1Z_3Z_4^2+2Z_1Z_2Z_4^2+2Z_2Z_3^2Z_4+2Z_2^2Z_3Z_4)\Big\}\\
&=\frac{2ig^2\langle 3,4\rangle\langle 2,4\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}\Big\{\frac{1}{2}\tilde{\chi}_4^A\chi_{1A}\phi_3^{A'B'}\phi_{2A'B'}\langle 1,2\rangle\langle 3,4\rangle+\tilde{\chi}_4^{A'}\phi_{3A'B'}\phi_2^{B'A}\chi_{1A}\langle 2,3\rangle\langle 4,1\rangle\Big\},
\end{aligned}
\tag{5.11}
$$

where the identities (A.15) and (A.17) are used. The result agrees with (3.4).

5.2 Extended MHV amplitudes at $\mathcal{O}(M)$

In the cases of extended MHV amplitudes at $\mathcal{O}(M)$, the external functions $f_i(\Psi_i^A)$ altogether have 6 fermionic coordinates Ψ's. In order to have 8 θ's for the integrand in (5.7), the mass term has to contribute exactly once. Therefore, the resulting amplitudes are of the order $\mathcal{O}(M)$.

5.2.1 $A_{\mathcal{O}(M)}(+1/2,+1,-1,+1/2)$

The external wavefunctions are $\Psi_1^A\chi_{1A}$, A_2, $\frac{1}{24}\epsilon_{ABCD}\Psi_3^A\Psi_3^A\Psi_3^A\Psi_3^DG_3$, and $\Psi_4^A\chi_{4A}$. The

amplitude by (5.7) is thus

$$
\begin{aligned}
&2ig^2\int\prod_i^4 d^4\Psi_i^A\int d^8\theta_a^A\prod_{i=1}^4\delta^4(\Psi_i^A+\theta_1^A+\theta_2^A Z_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^E Z_i^2)\\
&\quad\times\frac{1}{\prod_{i=1}^4\langle i,i+1\rangle}(\Psi_1^A\chi_{1A})(A_2)(\frac{1}{24}\epsilon_{A''B''C''D''}\Psi_3^{A''}\Psi_3^{B''}\Psi_3^{C''}\Psi_3^{D''}G_3)(\Psi_4^{A'''}\chi_{4A'''})\\
&=-2ig^2\int d^8\theta_a^A\frac{(Z_1Z_4+Z_3^2-Z_3Z_4-Z_3Z_1)(Z_1-Z_4)}{\prod_{i=1}^4\langle i,i+1\rangle}\\
&\quad\times M^{AA'''}\chi_{1A}A_2G_3\chi_{4A'''}\frac{1}{36}\epsilon_{A''FGH}\theta_1^{A'}\theta_1^F\theta_1^G\theta_1^H\epsilon_{IJKL}\theta_2^I\theta_2^J\theta_2^K\theta_2^L\\
&=2ig^2\frac{\langle 3,1\rangle\langle 3,4\rangle\langle 4,1\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}\chi_{4B}M^{BA}\chi_{1A}G_3A_2. \qquad (5.12)
\end{aligned}
$$

The result concurs with (3.5).

5.2.2 $A_{\mathcal{O}(M)}(+1/2,+1/2,-1/2,+1/2)$

The external wavefunctions are $\Psi_1^A\chi_{1A}$, $\Psi_2^A\chi_{2A}$, $\frac{1}{6}\epsilon_{ABCD}\Psi_3^A\Psi_3^A\Psi_3^A\tilde{\chi}_3^D$ and $\Psi_4^A\chi_{4A}$. The integral (5.7) gives the amplitude

$$
\begin{aligned}
&2ig^2\int\prod_i^4 d^4\Psi_i^A\int d^8\theta_a^A\prod_{i=1}^4\delta^4(\Psi_i^A+\theta_1^A+\theta_2^A Z_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^E Z_i^2)\\
&\quad\times\frac{1}{\prod_1^4\langle i,i+1\rangle}(\Psi_1^A\chi_{1A})(\Psi_2^{A'}\chi_{2A'})(\frac{1}{6}\epsilon_{A''B''C''D''}\Psi_3^{A''}\Psi_3^{B''}\Psi_3^{C''}\tilde{\chi}_3^{D''})(\Psi_4^{A'''}\chi_{4A'''})\\
&=\frac{2ig^2}{\prod_1^4\langle i,i+1\rangle}\Big\{\chi_{1A}M^{AA'}\chi_{2A'}\tilde{\chi}^{3A''}\chi_{4A''}\Big(Z_1Z_2(Z_1-Z_2)+Z_3(Z_2^2-Z_1^2)+Z_3^2(Z_1-Z_2)\Big)\\
&\quad+\chi_{1A}\tilde{\chi}^{3A}\chi_{2A'}M^{A'A'''}\chi_{4A'''}\Big(Z_1Z_4(Z_4-Z_1)+Z_3(Z_1^2-Z_4^2)+Z_3^2(Z_4-Z_1)\Big)\\
&\quad+\chi_{2A'}\tilde{\chi}^{3A'}\chi_{1A}M^{AA'''}\chi_{4A'''}\Big(Z_2Z_4(Z_4-Z_2)+Z_3(Z_2^2-Z_4^2)+Z_3^2(Z_4-Z_2)\Big)\Big\}\\
&=\frac{2ig^2}{\prod_1^4\langle i,i+1\rangle}\Big\{\chi_{1A}M^{AA'}\chi_{2A'}\tilde{\chi}^{3A''}\chi_{4A''}\langle 1,2\rangle\langle 2,3\rangle\langle 3,1\rangle+\chi_{1A}\tilde{\chi}^{3A}\chi_{2A'}M^{A'A'''}\\
&\qquad\times\chi_{4A'''}\langle 2,4\rangle\langle 2,3\rangle\langle 3,4\rangle+\chi_{2A'}\tilde{\chi}^{3A'}\chi_{1A}M^{AA'''}\chi_{4A'''}\langle 4,1\rangle\langle 1,3\rangle\langle 3,4\rangle\Big\}, \qquad (5.13)
\end{aligned}
$$

which agrees with (3.6).

5.2.3 $A_{\mathcal{O}(M)}(+1/2,0,0,+1/2)$

The external wavefunctions are given by $\Psi_1^A\chi_{1A}$, $\frac{1}{2!}\Psi_2^A\Psi_2^B\phi_{2AB}$, $\frac{1}{2!}\Psi_3^A\Psi_3^B\phi_{3AB}$ and $\Psi_1^A\chi_{1A}$. The amplitude is

$$
\begin{aligned}
&2ig^2\int\prod_i^4 d^4\Psi_i^A\int d^8\theta_a^A\prod_{i=1}^4\delta^4(\Psi_i^A+\theta_1^A+\theta_2^A Z_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^E Z_i^2)\\
&\quad\times\frac{1}{\prod_{i=1}^4\langle i,i+1\rangle}(\Psi_1^A\chi_{1A})(\frac{1}{2!}\Psi_2^{A'}\Psi_2^{B'}\phi_{2A'B'})(\frac{1}{2!}\Psi_3^{A''}\Psi_3^{B''}\phi_{3A''B''})(\Psi_1^{A'''}\chi_{1A'''})
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ig^2}{2\prod_{i=1}^4\langle i,i+1\rangle}\Big\{\chi_{4A'''}M^{A'''A}\chi_{1A}\phi_3^{BC}\phi_{2BC}\\
&\qquad\qquad\times\Big(Z_1^2Z_4 - Z_1Z_4^2 + Z_2Z_4^2 - Z_2^2Z_4Z_1Z_3^2 - Z_1^2Z_3\Big)\\
&\qquad +2\chi_{4A'''}M^{A'''A'}\phi_{3A'B'}\phi_2^{B'A}\chi_{1A}\Big(Z_2Z_4^2 - Z_2^2Z_4 + Z_3^2Z_4 - Z_3Z_4^2\Big)\\
&\qquad +2\chi_{4A'''}\phi_3^{A'''B'}\phi_{2B'A'}M^{A'A}\chi_{1A}\Big(Z_1^2Z_2 - Z_1Z_2^2 + Z_1Z_3^2 - Z_1^2Z_3\Big)\\
&\qquad +2\epsilon^{AB'B''A'''}M^{A''A'}\chi_{4A'''}\chi_{1A}\phi_3^{A''B''}\phi_{2A'B'}\Big(Z_2Z_3^2 - Z_2^2Z_3\Big)\Big\}\\
&= \frac{ig^2}{2\prod_{i=1}^4\langle i,i+1\rangle}\Big\{\chi_{4A'''}M^{A'''A}\chi_{1A}\phi_3^{BC}\phi_{2BC}\Big(\langle1,2\rangle\langle2,3\rangle\langle3,4\rangle + \langle1,2\rangle\langle3,4\rangle\langle1,4\rangle\Big)\\
&\qquad +2\chi_{4A'''}M^{A'''A'}\phi_{3A'B'}\phi_2^{B'A}\chi_{1A}\langle2,3\rangle\langle3,4\rangle\langle4,2\rangle\\
&\qquad +2\chi_{4A'''}\phi_3^{A'''B'}\phi_{2B'A'}M^{A'A}\chi_{1A}\langle1,2\rangle\langle2,3\rangle\langle3,1\rangle\Big\}\,,
\end{aligned}
\tag{5.14}
$$

where the identities (A.15), (A.19) and $M^{A'A''}\phi_{2A'A''}=0$ (due to the symmetry of M and antisymmetry of ϕ) are used. The result concurs with (3.7).

5.2.4 $A_{\mathcal{O}(M)}(0,0,0)$

Finally, we study the simple but instructive case: 3-scalar amplitude, i.e., $A_{\mathcal{O}(M)}(0,0,0)$. For the amplitudes of 3 massless particles, the momentum conservation implies that p_i are collinear. Angular momentum conservation further forces the amplitude to vanish for 3-gluon scattering. For 3-scalar scattering, this is not the case and in fact this is the only nonvanishing 3-particle extended MHV of $\mathcal{O}(M)$. The amplitude can be obtained as well from the integral in super-twistor space [by a formula similar to (5.7) but with only 3 external functions and the prefactor $2g^2$ replaced by $g/2$]. It is

$$
\begin{aligned}
&\frac{ig}{2}\int\prod_i^3 d^4\Psi_i^A\int d^8\theta_a^A\prod_{i=1}^3\delta^4(\Psi_i^A+\theta_1^A+\theta_2^AZ_i-\frac{1}{6}M^{AB}\epsilon_{BCDE}\theta_2^C\theta_2^D\theta_2^EZ_i^2)\\
&\quad\times\frac{1}{\prod_{i=1}^3\langle i,i+1\rangle}\left(\frac{1}{2!}\Psi_1^A\Psi_1^B\phi_{1AB}\right)\left(\frac{1}{2!}\Psi_2^{A'}\Psi_2^{B'}\phi_{2A'B'}\right)\left(\frac{1}{2!}\Psi_3^{A''}\Psi_3^{B''}\phi_{3A''B''}\right)\\
&=\frac{ig}{2\prod_{i=1}^3\langle i,i+1\rangle}\Big\{Z_1^2Z_2(M^{AA'}\phi_{1AB}\phi_{2A'B'}\phi_3^{BB'})+Z_1^2Z_3(M^{AA'}\phi_{1AB}\phi_{3A'B'}\phi_2^{BB'})\\
&\qquad+Z_2^2Z_3(M^{AA'}\phi_{2AB}\phi_{3A'B'}\phi_1^{BB'})+Z_2^2Z_1(M^{AA'}\phi_{2AB}\phi_{1A'B'}\phi_3^{BB'})\\
&\qquad+Z_3^2Z_1(M^{AA'}\phi_{3AB}\phi_{1A'B'}\phi_2^{BB'})+Z_3^2Z_2(M^{AA'}\phi_{3AB}\phi_{2A'B'}\phi_1^{BB'})\Big\}\\
&=\frac{ig(M^{AA'}\phi_{1AB}\phi_{2A'B'}\phi_3^{BB'})}{2\prod_{i=1}^3\langle i,i+1\rangle}\left\{Z_1^2Z_2-Z_1^2Z_3+Z_2^2Z_3-Z_2^2Z_1+Z_3^2Z_1-Z_3^2Z_2\right\}\\
&=-\frac{ig}{2}\left(M^{AA'}\phi_{1AB}\phi_{2A'B'}\phi_3^{BB'}\right),
\end{aligned}
\tag{5.15}
$$

and we used the identities (A.17) and $M^{AB}\phi_{iAB}=0$.

This result agrees with the tree-level Feynman diagram calculation. At tree level, we have only one diagram as in figure 4, where the Feynman rule for the vertex together

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with $\varphi_{\mathcal{I}}$, $\varphi_{\mathcal{J}}$ and $\varphi_{\mathcal{K}}$ for the external legs trivially yields the same result. This confirms again that the mass-deformed 3ϕ-interaction discussed in section 2.2 and appendix B.6 is required to have the correct holomorphic structure in super-twistor space in the presence of the chiral mass deformation.

6. $N = 1$ Supersymmetry

The chiral-mass deformation that we studied in section 2-section 5 depends on 10 complex parameters M^{AB} $(A, B = 1, \ldots, 4)$ and in general breaks $N = 4$ supersymmetry completely. In this section we will study a subset of these deformations – those that preserve $N = 1$ supersymmetry.

The unperturbed $N = 4$ super Yang-Mills theory has 8 supersymmetry generators $Q_{\alpha A}$ and $\overline{Q}^{A}_{\dot\alpha}$. The $N = 1$ deformation that we will study in this section will preserve the generators with $A = 4$ and break those with $A = 1, 2, 3$. We will use indices $i, j, k, \cdots = 1, 2, 3$ instead of A, when the summation excludes $A = 4$.

In $N = 1$ superspace notation the theory contains a vector multiplet V with associated chiral field strength multiplet W_α and its complex conjugate $\overline{W}^{\dot\alpha}$. In addition there are 3 chiral multiplets Φ^i and their complex conjugates $\overline{\Phi}_i$, where $i = 1, 2, 3$ is an $SU(3)$ flavor index. The Lagrangian is given by

$$\begin{aligned} 2{g_{YM}}^2\mathcal{L} = &\int d^2\overline{\theta}d^2\theta\ \mathrm{tr}\Big\{\overline{\Phi}_i e^V \Phi^i\Big\} + \int d^2\theta\,\mathrm{tr}\Big\{W_\alpha W^\alpha + \epsilon_{ijk}\Phi^i[\Phi^j,\Phi^k]\Big\} \\ &+ \int d^2\overline{\theta}\,\mathrm{tr}\Big\{\overline{W}^{\dot\alpha}\overline{W}_{\dot\alpha} + \epsilon^{ijk}\overline{\Phi}_i[\overline{\Phi}_j,\overline{\Phi}_k] + M^{ij}\overline{\Phi}_i\overline{\Phi}_j\Big\} \end{aligned} \tag{6.1}$$

Note that the chiral mass term only deforms the $d^2\overline{\theta}$ integral, and the chiral and anti-chiral superpotentials are not the complex conjugates of each other! (The situation is reminiscent of the deformations used in [24].)

Integrating out the auxiliary fields in the superfields, we find that the mass deformation adds the following extra terms to the potential

$$\Delta U = \frac{1}{2}\,\mathrm{tr}\Big\{M^{ij}\overline{\psi}_{\dot\alpha i}\overline{\psi}^{\dot\alpha}_j + g_{YM}M^{ij}\epsilon_{jkl}\phi^*_i[\phi^k,\phi^l]\Big\}, \tag{6.2}$$

where ϕ_i is the $\theta = \overline{\theta} = 0$ component of Φ_i/g_{YM}.

Now let us turn to twistor space $\mathbb{CP}^{3|4}$. Let us first identify the action of the $N = 1$ supersymmetry generators on the undeformed twistor space. It is

$$\delta\lambda_\alpha = 0, \qquad \delta\mu_{\dot\alpha} = \overline{\zeta}_{\dot\alpha}\Theta^4, \qquad \delta\Theta^i = 0 \quad (i = 1, 2, 3), \qquad \delta\Theta^4 = \zeta_\alpha\lambda^\alpha,$$

where ζ and $\overline{\zeta}$ are the anti-commuting SUSY parameters. Using the holomorphic coordinates on patch U from section 4, we can rewrite the SUSY transformations as

$$\delta X = \overline{\zeta}_{\dot 1}\Psi^4, \qquad \delta Y = \overline{\zeta}_{\dot 2}\Psi^4, \qquad \delta Z = 0, \qquad \delta\Psi^i = 0, \quad (i = 1, 2, 3), \qquad \delta\Psi^4 = \zeta_1 + \zeta_2 Z. \tag{6.3}$$

The mass-deformed space, given by the transition functions (4.7), which can be written in our case as

$$Z' = \frac{1}{Z}, \quad X' = \frac{X}{Z}, \quad Y' = \frac{Y}{Z}, \quad \Psi'^i = \frac{1}{Z}\Psi^i + \frac{1}{2Z^2}M^{ij}\epsilon_{jkl}\Psi^k\Psi^l\Psi^4, \quad \Psi'^4 = \frac{1}{Z}\Psi^4, \quad (6.4)$$

is not invariant under the same SUSY transformations (6.3), because they would imply

$$\delta\Psi'^i = \frac{1}{2Z^2}M^{ij}\epsilon_{jkl}\Psi^k\Psi^l(\zeta_1 + \zeta_2 Z) = \frac{1}{2}M^{ij}\epsilon_{jkl}\Psi'^k\Psi'^l\left(\zeta_1 + \frac{1}{Z'}\zeta_2\right),$$

which is ill-defined near $Z' = 0$. We can fix this by modifying the SUSY transformation (6.3) to

$$\delta X = \overline{\zeta}_{\dot{1}}\Psi^4, \quad \delta Y = \overline{\zeta}_{\dot{2}}\Psi^4, \quad \delta Z = 0, \quad \delta\Psi^i = -\tfrac{1}{2}\zeta_2 M^{ij}\epsilon_{jkl}\Psi^k\Psi^l, \quad \delta\Psi^4 = \zeta_1 + \zeta_2 Z. \quad (6.5)$$

This modified transformation law still satisfies the correct commutation relations, as can be seen after some algebra and using $M^{ij} = M^{ji}$. Thus, the deformed twistor space associated with (6.1) is indeed supersymmetric.

7. Summary and discussion

In this paper we studied a new deformation of twistor string theory; we tested the proposal that the deformation of twistor space to a space whose complex structure is defined by the transition functions (6.4) is associated with the deformation of $N = 4$ super Yang-Mills theory given by the following Lagrangian:

$$g^2\mathcal{L} = \frac{1}{4}\,\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu} + 2D_\mu\phi_{\mathcal{I}}D^\mu\phi_{\mathcal{I}} - [\phi_{\mathcal{I}},\phi_{\mathcal{J}}]^2\right) + \frac{i}{2}\,\mathrm{tr}\left(\overline{\psi}\gamma^\mu D_\mu\psi + i\overline{\psi}\Gamma^{\mathcal{I}}[\phi_{\mathcal{I}},\psi]\right)$$
$$+ \frac{i}{2}\,\mathrm{tr}\left(M^{AB}\overline{\psi}^{\dot{\alpha}}_A\overline{\psi}_{\dot{\alpha}B} + \frac{1}{4}M^{\mathcal{I}\mathcal{J}\mathcal{K}}\phi_{\mathcal{I}}[\phi_{\mathcal{J}},\phi_{\mathcal{K}}]\right).$$

Here $M^{AB} = M^{BA}$ is the mass parameter in the representation **10** of the R-symmetry group $SU(4)$, and $M^{\mathcal{I}\mathcal{J}\mathcal{K}}$ is linearly related to M^{AB} and is given in (A.20).

We calculated tree-level 4-point scattering amplitudes up to order $O(M)$ and we checked that these amplitudes can be reproduced from an integral over a moduli space of holomorphic curves in the deformed twistor space, just like the undeformed case.

Among other things, twistor string theory is interesting in that it opens a window into the nonperturbative aspects of topological string theory on supermanifolds. There has been a lot of progress recently in understanding the nonperturbative aspects of topological string theories on ordinary manifolds (see for instance [25]-[31]).

The perturbative open topological string theory with target space $\mathbb{CP}^{3|4}$ reproduces a self-dual truncation of $N = 4$ SYM theory [3]. Extensions to other weighted projective target spaces were demonstrated in [20]-[23]. It was also suggested in [3] that D1-instantons in the topological string theory complete the self-dual truncation to a full $N = 4$ SYM theory. In fact, the integral (5.1) (copied from [3]) is the one-instanton contribution to the amplitude. Our results suggest a possible extension of these ideas to a 10-parameter family of deformations of the target space $\mathbb{CP}^{3|4}$.

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Other deformations of twistor string theory have been studied in [32][33], and orbifolds of twistor string theory were studied in [34][35]. For example, Kulaxizi and Zoubos [32] translated the so-called β-deformations of $N = 4$ SYM [36]-[40] into a non-anticommutativity among the fermionic coordinates of super twistor space. It would be interesting to add a chiral mass term to these deformations and to the orbifold constructions.

Another possible direction for further study is the reduction to $D = 3$ and lower dimensions. The relevant target space for $D = 3$ is the weighted projective space $W\mathbb{CP}^{2|1,1,1,1}$. This reduction was studied in [1][41][42] and involves minitwistor space [43][44]. Other reductions of twistor string theory have been recently proposed in [45]. It would be interesting to further study the corresponding reduction of the complex structure deformation that was described in the present paper.

Acknowledgments

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A. Notation and useful formulae

A.1 Spinors

Our metric is in Minkowski signature $(+,-,-,-)$. Spinor indices of type $(1/2,0)$ and $(0,1/2)$ are raised and lowered with antisymmetric tensors $\epsilon_{\alpha\beta}$, $\epsilon_{\dot\alpha\dot\beta}$ and their inverses $\epsilon^{\alpha\beta}$, $\epsilon_{\dot\alpha\dot\beta}$:

$$\lambda_\alpha = \epsilon_{\alpha\beta}\lambda^\beta, \qquad \lambda^\alpha = \epsilon^{\alpha\beta}\lambda_\beta, \qquad \tilde\lambda_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\tilde\lambda^{\dot\beta}, \qquad \tilde\lambda^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\tilde\lambda_{\dot\beta}, \tag{A.1}$$

with

$$\epsilon_{\alpha\beta} = \epsilon_{\dot\alpha\dot\beta} = -\epsilon^{\alpha\beta} = -\epsilon^{\dot\alpha\dot\beta}, \qquad \epsilon_{12} = 1. \tag{A.2}$$

The Lorentz invariants $\langle\lambda_1,\lambda_2\rangle$ and $[\tilde\lambda_1,\tilde\lambda_2]$ are defined as

$$\langle\lambda_1,\lambda_2\rangle = -\langle\lambda_2,\lambda_1\rangle = \epsilon_{\alpha\beta}\lambda_1^\alpha\lambda_2^\beta = \lambda_1^\alpha\lambda_{2\alpha} = -\lambda_{1\alpha}\lambda_2^\alpha, \tag{A.3}$$

and

$$[\tilde\lambda_1,\tilde\lambda_2] = -[\tilde\lambda_2,\tilde\lambda_1] = \epsilon^{\dot\alpha\dot\beta}\tilde\lambda_{1\dot\alpha}\tilde\lambda_{2\dot\beta} = \tilde\lambda_{1\dot\alpha}\tilde\lambda_2^{\dot\alpha} = -\tilde\lambda_1^{\dot\alpha}\tilde\lambda_{2\dot\alpha}. \tag{A.4}$$

The vector representation of $SO(3,1)$ can be represented as the tensor product of two spinor representation of opposite chirality via:

$$p_{\alpha\dot\alpha} = \sigma^\mu_{\alpha\dot\alpha}p_\mu, \qquad p^{\dot\alpha\alpha} = \overline{\sigma}^{\mu\dot\alpha\alpha}p_\mu, \tag{A.5}$$

where $\sigma^\mu = (\mathbf{1},\vec\sigma)$, $\overline{\sigma}^\mu = (\mathbf{1},-\vec\sigma)$ and $\vec\sigma$ are Pauli matrices.

Some useful formulae for σ-matrices are listed below:

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma_{\mu\beta\dot{\beta}} = 2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}, \qquad \overline{\sigma}^{\dot{\alpha}\alpha}_{\mu}\sigma^{\mu}_{\beta\dot{\beta}} = 2\delta^{\alpha}_{\beta}\delta^{\dot{\alpha}}_{\dot{\beta}}, \tag{A.6}$$

$$(\sigma^{\mu}\overline{\sigma}^{\nu} + \sigma^{\nu}\overline{\sigma}^{\mu})_{\alpha}{}^{\beta} = 2\eta^{\mu\nu}\delta_{\alpha}{}^{\beta}, \qquad (\overline{\sigma}^{\mu}\sigma^{\nu} + \overline{\sigma}^{\nu}\sigma^{\mu})_{\dot{\alpha}}{}^{\dot{\beta}} = 2\eta^{\mu\nu}\delta^{\dot{\alpha}}{}_{\dot{\beta}}. \tag{A.7}$$

and

$$\mathrm{tr}\,\sigma^{\mu}\overline{\sigma}^{\nu} = 2\eta^{\mu\nu}. \tag{A.8}$$

The inner product of two vectors gives

$$W_{\mu}V^{\mu} = \eta^{\mu\nu}W_{\mu}V_{\nu} = \frac{1}{2}\,\mathrm{tr}[\sigma^{\mu}\overline{\sigma}^{\nu}]W_{\mu}V_{\nu} = \frac{1}{2}W_{\alpha\dot{\alpha}}V^{\dot{\alpha}\alpha} = \frac{1}{2}W^{\dot{\alpha}\alpha}V_{\alpha\dot{\alpha}}. \tag{A.9}$$

If p^{μ} is lightlike, we can decompose p^{μ} as

$$p_{\alpha\dot{\alpha}} = \lambda_{\alpha}\tilde{\lambda}_{\dot{\alpha}}. \tag{A.10}$$

Furthermore, if q^{μ} is also lightlike (written as $q_{\alpha\dot{\alpha}} = \mu_{\alpha}\tilde{\mu}_{\dot{\alpha}}$) we have

$$p \cdot q = -\frac{1}{2}\langle\lambda,\mu\rangle[\tilde{\lambda},\tilde{\mu}]. \tag{A.11}$$

A.2 $SU(4)$ R-symmetry indices

In $N = 4$ super Yang-Mills theory, the scalar field $\phi_{\mathcal{I}}$ is real and in the representation **6** of $SU(4)$. Since **6** is the antisymmetric part of $\mathbf{4}\times\mathbf{4}$ or $\bar{\mathbf{4}}\times\bar{\mathbf{4}}$, we can exchange $\phi_{\mathcal{I}}$ ($\mathcal{I}$ is an index of **6**) for the complex antisymmetric field $\phi_{AB} = -\phi_{BA}$ (A, B: indices of **4**) or $\phi^{AB} = -\phi^{BA}$ (A, B: indices of $\bar{\mathbf{4}}$) by

$$\phi_{AB} = \Gamma^{\mathcal{I}}_{AB}\phi_{\mathcal{I}}, \qquad \phi^{AB} = \Gamma^{\mathcal{I}AB}\phi_{\mathcal{I}}, \tag{A.12}$$

where $\Gamma^{\mathcal{I}}$'s satisfy

$$\delta_{\mathcal{I}\mathcal{J}}\Gamma^{\mathcal{I}AB}\Gamma^{\mathcal{J}}_{CD} = \frac{1}{2}(\delta^{A}_{C}\delta^{B}_{D} - \delta^{A}_{D}\delta^{B}_{C}), \qquad \delta_{\mathcal{I}\mathcal{J}}\Gamma^{\mathcal{I}}_{AB}\Gamma^{\mathcal{J}}_{CD} = \frac{1}{2}\epsilon_{ABCD}, \tag{A.13}$$

and

$$\Gamma^{\mathcal{I}AB} = \frac{1}{2}\,\epsilon^{ABCD}\Gamma^{\mathcal{I}}_{CD}, \qquad \Gamma^{\mathcal{I}}_{AB} = \frac{1}{2}\,\epsilon_{ABCD}\Gamma^{\mathcal{I}CD}. \tag{A.14}$$

The reality condition on ϕ now reads as

$$\phi^{AB} = \frac{1}{2}\,\epsilon^{ABCD}\phi_{CD}, \qquad \phi_{AB} = \frac{1}{2}\,\epsilon_{ABCD}\phi^{CD}, \qquad \text{or} \quad (\phi_{AB})^{*} = \phi^{AB}. \tag{A.15}$$

It follows that

$$\phi^{\mathcal{I}}_{1}\phi_{2\mathcal{I}} = \phi^{\mathcal{I}}_{2}\phi_{1\mathcal{I}} = \delta^{\mathcal{I}J}\phi_{1\mathcal{I}}\phi_{2\mathcal{J}} = \phi_{1}^{AB}\phi_{2AB}. \tag{A.16}$$

Some useful formulae regarding antisymmetry of ϕ_{AB} are listed below:

$$-\frac{1}{2}\,\delta^{A}_{B}\,\phi_{2CD}\phi_{3}^{CD} = \phi_{2}^{AC}\phi_{3CB} + \phi_{3}^{AC}\phi_{2CB}, \tag{A.17}$$

$$-\frac{1}{2}M^{AB}\phi_2^{CD}\phi_{3CD} = M^{BC}\left(\phi_{2CD}\phi_3^{DA}+\phi_{3CD}\phi_2^{DA}\right) = M^{AC}\left(\phi_{2CD}\phi_3^{DB}+\phi_{3CD}\phi_2^{DB}\right), \tag{A.18}$$

and

$$\epsilon^{AB'B''A'''}\phi_3^{A''B''}\phi_{2A'B'} = -\delta^A_{A''}\phi_3^{B'A'''}\phi_{2A'B'}+\delta^{A'''}_{A''}\phi_3^{B'A}\phi_{2A'B'}-\phi_3^{A'''A}\phi_{2A'A''}. \tag{A.19}$$

The mass parameter $M^{AB} = M^{BA}$ is in the irreducible representation **10** of the R-symmetry group $SU(4)$. Using the double cover $SU(4) \to SO(6)$, the representation **10** of $SU(4)$ is induced from an irreducible representation of $SO(6)$ which can be realized as self-dual anti-symmetric 3-tensors. Explicitly, define

$$M^{\mathcal{IJK}} := \Gamma^{[\mathcal{I}}_{AB}\Gamma^{\mathcal{J}}_{CD}\Gamma^{\mathcal{K}]}_{EF}\epsilon^{ABCE}M^{DF} \Longrightarrow M^{\mathcal{IJK}} = \tfrac{1}{3!}\epsilon_{\mathcal{IJKPQR}}M^{\mathcal{PQR}}. \tag{A.20}$$

Then, the 3-ϕ coupling from (B.5) can be written as

$$M^{AB}\epsilon^{CDEF}\,\mathrm{tr}\Big\{\phi_{AC}[\phi_{BD},\phi_{EF}]\Big\} = M^{\mathcal{IJK}}\,\mathrm{tr}\Big\{\phi_{\mathcal{I}}\phi_{\mathcal{J}}\phi_{\mathcal{K}}\Big\}. \tag{A.21}$$

We define the $su(4)$-invariant symbol

$$\Gamma^{\mathcal{IJK}}_{AB} := \Gamma^{[\mathcal{I}}_{DE}\Gamma^{\mathcal{J}}_{CA}\Gamma^{\mathcal{K}]}_{FB}\epsilon^{CDEF}. \tag{A.22}$$

It is symmetric in AB and anti-symmetric in $\mathcal{IJK}$ and satisfies the self-duality relation

$$\Gamma^{\mathcal{IJK}}_{AB} = \frac{1}{3!}\epsilon^{\mathcal{IJK}}{}_{\mathcal{PQR}}\Gamma^{\mathcal{PQR}}_{AB}. \tag{A.23}$$

We can then write

$$M^{\mathcal{IJK}} \equiv \Gamma^{\mathcal{IJK}}_{AB}M^{AB}. \tag{A.24}$$

B. Feynman rules with chiral mass terms

The Lagrangian of $D = 4$, $N = 4$ Yang-Mills theory is given by

$$\mathcal{L} = \frac{1}{4g^2}\,\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}+2D_\mu\phi_{\mathcal{I}}D^\mu\phi_{\mathcal{I}}-[\phi_{\mathcal{I}},\phi_{\mathcal{J}}]^2\right)+\frac{i}{2g^2}\,\mathrm{tr}\left(\overline{\psi}\gamma^\mu D_\mu\psi+i\overline{\psi}\Gamma^{\mathcal{I}}[\phi_{\mathcal{I}},\psi]\right), \tag{B.1}$$

where $\overline{\psi}^a = (\psi^\alpha, \overline{\psi}_{\dot\alpha})$ is a Dirac spinor, and we treat ψ^α and $\overline{\psi}_{\dot\alpha}$ independently.[5]

In terms of $N = 1$ superfields the Lagrangian is

$$\begin{aligned} g^2\mathcal{L} = &\int d^2\overline{\theta}d^2\theta\;\mathrm{tr}\Big\{\overline{\Phi}_i e^V\Phi^i\Big\}+\int d^2\theta\,\mathrm{tr}\Big\{W_\alpha W^\alpha+\epsilon_{ijk}\Phi^i[\Phi^j,\Phi^k]\Big\}\\ &+\int d^2\overline{\theta}\,\mathrm{tr}\Big\{\overline{W}^{\dot\alpha}\overline{W}_{\dot\alpha}+\epsilon^{ijk}\overline{\Phi}_i[\overline{\Phi}_j,\overline{\Phi}_k]\Big\},\\ &\text{chiral superfields:}\qquad \Phi_i = \phi_i+\sqrt{2}\theta\psi_i+\theta\theta F_i,\quad i=1,2,3 \end{aligned}$$

[5] In $N = 4$ SYM ψ_a is actually a Majorana spinor. But in order to get the Feynman rules by analogy with ordinary QED, we treat the two chiralities independently and in the end identify external fermions as anti-fermions to take into account that ψ is Majorana. See also appendix B.5.

$$\text{vector superfield:} \qquad V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D, \tag{B.2}$$

where we identify the component fields of (B.2) with those of (B.1) according to

$$\phi_{A=i,B=j} = \epsilon_{ijk}\phi^k, \qquad \phi_{A=4,B=i} = \phi_i^*, \tag{B.3}$$

and

$$(\psi_{A=i}, \overline{\psi}_{A=i}) = (\psi_i, \overline{\psi}_i), \qquad (\psi_{A=4}, \overline{\psi}_{A=4}) = (\lambda, \bar{\lambda}), \tag{B.4}$$

where $\phi^{AB} = -\phi^{BA} = \Gamma^{\mathcal{I}AB}\phi_{\mathcal{I}} = \epsilon^{ABCD}\phi_{CD}/2$ (as defined in appendix A.2).

Now, if the chiral mass term $M^{ij}\overline{\Phi}_i\overline{\Phi}_j$ is added to (B.2) as discussed in (6.1), $N = 4$ supersymmetry is broken to $N = 1$ and the extra term (6.2) leads to

$$\Delta U = \mathrm{tr}\left\{ M^{AB}\overline{\psi}_{\dot{\alpha}A}\overline{\psi}^{\dot{\alpha}}_B + \frac{1}{4}\, g M^{AB}\epsilon^{CDEF}\phi_{AC}[\phi_{BD}, \phi_{EF}] \right\}, \tag{B.5}$$

which is added to (B.1) (with $M^{A=4,B} = M^{B,A=4} = 0$).[6]

In this paper (unless otherwise mentioned), we considered the general chiral mass term (i.e., $M^{4A} = M^{A4}$ could be nonzero) and the mass deformation had the form (B.5) (thus breaking $N = 1$ supersymmetry in general). In the following, we first present the Feynman rules involving the chiral spinor mass term in B.1–B.5 and later in B.6 we present the Feynman rule for the 3ϕ-interaction.

B.1 Fermion propagators

When the chiral spinor mass term $M^{AB}\overline{\psi}_{\dot{\alpha}A}\overline{\psi}^{\dot{\alpha}}_B$ is added to (B.1), the Dirac part of the modified Lagrangian reads (the color group factor is ignored)

$$\mathcal{L}_{Dirac} = \overline{\psi}(i\gamma^\mu\partial_\mu)\psi - M^{AB}\overline{\psi}_{\dot{\alpha}A}\overline{\psi}^{\dot{\alpha}}_B = (\psi^\alpha_A, \overline{\psi}_{\dot{\alpha}A}) \begin{pmatrix} 0 & \delta_A{}^B p_\mu \sigma^\mu_{\alpha\dot{\beta}} \\ \delta^A{}_B\, p_\mu \overline{\sigma}^{\mu\dot{\alpha}\beta} & -M^{AB}\delta^{\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix} \begin{pmatrix} \psi_{\beta B} \\ \overline{\psi}^{\dot{\beta}}_B \end{pmatrix} \tag{B.6}$$

The spinor propagator is $i\times$(inverse of the middle operator) on the right of (B.6). With the identities in (A.7), the propagator is given by

$$\frac{i}{p^2}\begin{pmatrix} M^{AB}\delta_\alpha{}^\beta & \delta^A{}_B p_\mu \sigma^\mu_{\alpha\dot{\beta}} \\ \delta_A{}^B p_\mu \overline{\sigma}^{\mu\dot{\alpha}\beta} & 0 \end{pmatrix} = \frac{i}{p^2}\begin{pmatrix} M^{AB}\delta_\alpha{}^\beta & \delta^A{}_B p_{\alpha\dot{\beta}} \\ \delta_A{}^B p^{\dot{\alpha}\beta} & 0 \end{pmatrix}. \tag{B.7}$$

The corresponding Feynman rules are listed in figure 1.

B.2 Solutions of Dirac equation

With the anti-chiral mass term, the Dirac equation can be read off from (B.6) as

$$\begin{pmatrix} 0 & \delta_A{}^B p_\mu \sigma^\mu_{\alpha\dot{\beta}} \\ \delta^A{}_B p_\mu \overline{\sigma}^{\mu\dot{\alpha}\beta} & -M^{AB}\delta^{\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix} \begin{pmatrix} \psi_{\beta B} \\ \overline{\psi}^{\dot{\beta}}_B \end{pmatrix} = 0. \tag{B.8}$$

[6] The equality of the chiral spinor mass terms in (6.2) and in (B.5) is obvious, while the equality of the mass-deformed 3ϕ-interaction terms is less transparent and will be discussed in appendix B.6.

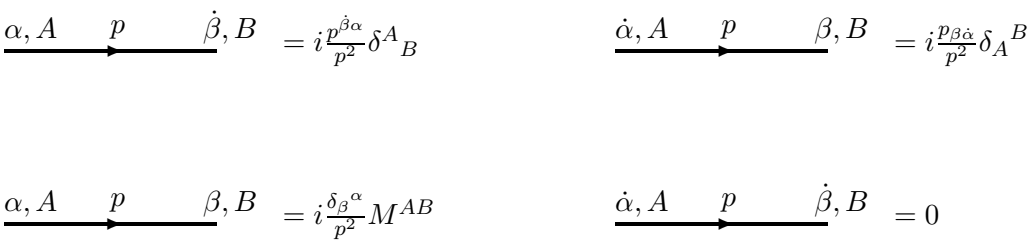

Figure 1: The fermion propagators in the presence of a chiral mass term.

The solutions were described in section 2.1. Consider the positive-frequency solutions, i.e., $\psi_\alpha(x) = \psi_\alpha(p)e^{-ip\cdot x}$ and $\overline{\psi}^{\dot{\alpha}}(x) = \overline{\psi}^{\dot{\alpha}}(p)e^{-ip\cdot x}$. These $\psi_\alpha(p)$ and $\overline{\psi}^{\dot{\alpha}}(p)$ obey the equation of motion (2.1). It is easy to see that $p^2 = p_{\alpha\dot{\alpha}}p^{\dot{\alpha}\alpha} = 0$, and thus the momentum is lightlike and can be decomposed as (2.2). A basis for the solutions is given by (2.3), which is invariant under

$$\eta'_\alpha \to \eta_\alpha + \zeta\lambda_\alpha, \qquad \varrho'^A \to \varrho^A - \zeta M^{AB}\widetilde{\varrho}_B, \qquad \widetilde{\varrho}'_B \to \widetilde{\varrho}_B \tag{B.9}$$

for any arbitrary number ζ.

B.3 Helicities and incoming functions

In the presence of an anti-chiral mass term, the helicity and chirality no longer coincide. However, since the 4-momentum p is still lightlike helicity is Lorentz invariant and can be used to specify the polarization of incoming and outgoing fermions.

When the lightlike p is written as (2.2) and if we treat $\widetilde{\varrho}_A(\lambda, \tilde{\lambda})$, $\varrho^A(\lambda, \tilde{\lambda})$ and $\eta_\alpha(\lambda, \tilde{\lambda})$ in the solution (2.3) as continuous functions of λ and $\tilde{\lambda}$, the helicity operator is given by

$$\hat{h} = \lambda_\alpha \frac{\partial}{\partial\lambda_\alpha} - \tilde{\lambda}_{\dot{\alpha}} \frac{\partial}{\partial\tilde{\lambda}_{\dot{\alpha}}}, \tag{B.10}$$

which gives eigenvalues $-2h$ when acting on the function $\psi(p)$ if $\psi(p)e^{ip\cdot x}$ is a momentum eigenstate [3]. To find the solutions of positive and negative helicities, we first study some properties of helicities:

Lemma B.1. *$\hat{h}f(E) = 0$ if $f(\lambda, \tilde{\lambda})$ is a function of the energy E only.*

Proof. $2E = (p_0 + p_3) + (p_0 - p_3) = p_{1\dot{1}} + p_{2\dot{2}} = \lambda_1\tilde{\lambda}_{\dot{1}} + \lambda_2\tilde{\lambda}_{\dot{2}}$. It is easy to show $\hat{h}E = 0$ and therefore $\hat{h}f(E) = 0$. □

Lemma B.2. *$\hat{h}\eta_\alpha = -\eta_\alpha$, if η_α is given by*[7]

$$\eta_1 = \frac{\lambda_2^*}{2E} = \frac{\tilde{\lambda}_{\dot{2}}}{2E}, \qquad \eta_2 = -\frac{\lambda_1^*}{2E} = -\frac{\tilde{\lambda}_{\dot{1}}}{2E}. \tag{B.11}$$

[7]The exact reason for the choice (B.11) will be clear when we study the normalization condition (B.15). The choice (B.11) satisfies (2.4). Also note that in Minkowski signature we have $\lambda_\alpha^* = \pm\tilde{\lambda}_{\dot{\alpha}}$ and we choose the + sign here for positive-frequency solutions (− sign is for negative-frequency solutions).

Proof. Follows immediately from $E = \lambda_1\tilde{\lambda}_{\dot{1}} + \lambda_2\tilde{\lambda}_{\dot{2}}$. □

Using (2.3), let's write the solution as a Dirac spinor[8]

$$\psi_a^A = \overline{\psi}_A^{\dot{\alpha}} + \psi_\alpha^A = \tilde{\lambda}^{\dot{\alpha}}\widetilde{\varrho}_A + \lambda_\alpha\varrho^A + M^{AB}\eta_\alpha\widetilde{\varrho}_B. \tag{B.12}$$

The eigenvalue problem $\hat{h}\psi_a = -2h\psi_a$ now becomes

$$\begin{aligned}\hat{h}\psi_a^A &= \hat{h}\left(\tilde{\lambda}^{\dot{\alpha}}\widetilde{\varrho}_A + \lambda_\alpha\varrho^A + M^{AB}\eta_\alpha\widetilde{\varrho}_B\right) \\ &= -\tilde{\lambda}^{\dot{\alpha}}\widetilde{\varrho}_A + \tilde{\lambda}^{\dot{\alpha}}(\hat{h}\widetilde{\varrho}_A) + \lambda_\alpha\varrho^A + \lambda_\alpha(\hat{h}\varrho^A) + M^{AB}\eta_\alpha(\hat{h}\widetilde{\varrho}_B) - M^{AB}\eta_\alpha\widetilde{\varrho}_B\end{aligned} \tag{B.13}$$

by Lemma B.2. Furthermore, by Lemma B.1, if we choose $\varrho = 0$ and $\widetilde{\varrho} = \widetilde{\varrho}(E)$, we get the positive-helicity state with $h = +1/2$; if we choose $\widetilde{\varrho} = 0$ and $\varrho = \varrho(E)$, we get the negative-helicity state with $h = -1/2$. Therefore,we have a basis of helicity states:

$$u_{+a}^A = \widetilde{\varrho}_A(E)\tilde{\lambda}^{\dot{\alpha}} + M^{AB}\widetilde{\varrho}_B(E)\eta_\alpha, \qquad u_{-a}^A = \varrho^A(E)\lambda_\alpha. \tag{B.14}$$

The normalization condition will fix ϱ, $\widetilde{\varrho}$ and η. Firstly, we consider the orthogonality of u^+ and u^-:

$$u_-^{\dagger A}u_+^B = (\varrho^{A*}\lambda_1^*, \varrho^{A*}\lambda_2^*, 0, 0)\begin{pmatrix} M^{BC}\widetilde{\varrho}_C\eta_1 \\ M^{BC}\widetilde{\varrho}_C\eta_2 \\ \widetilde{\varrho}_B\tilde{\lambda}^{\dot{1}} \\ \widetilde{\varrho}_B\tilde{\lambda}^{\dot{2}} \end{pmatrix} = \varrho^{A*}M^{BC}\widetilde{\varrho}_C\left(\lambda_1^*\eta_1 + \lambda_2^*\eta_2\right) = 0. \tag{B.15}$$

This together with (2.4) enforces the solution in (B.11).

Secondly, consider

$$u_-^{\dagger A}u_-^B = \varrho^{A*}\varrho^B(\lambda_1^*, \lambda_2^*, 0, 0)\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ 0 \\ 0 \end{pmatrix} = \varrho^{A*}\varrho^B\left(\lambda_1\tilde{\lambda}_{\dot{1}} + \lambda_2\tilde{\lambda}_{\dot{2}}\right) = 2E\varrho^{A*}\varrho^B. \tag{B.16}$$

To have the correct normalization condition, namely $u_-^{\dagger A}u_-^B = 2E\delta^{AB}$, ϱ^A's have to satisfy

$$\varrho^{A*}\varrho^B = \delta^{AB}. \tag{B.17}$$

Finally, we compute

$$\begin{aligned}u_+^{\dagger A}u_+^B &= \left((M^{AC}\widetilde{\varrho}_C\,\eta_1)^*, (M^{AC}\widetilde{\varrho}_C\,\eta_2)^*, (\widetilde{\varrho}_A\tilde{\lambda}^{\dot{1}})^*, (\widetilde{\varrho}_A\tilde{\lambda}^{\dot{2}})^*\right)\begin{pmatrix} M^{BD}\widetilde{\varrho}_D\eta_1 \\ M^{BD}\widetilde{\varrho}_D\eta_2 \\ \widetilde{\varrho}_B\tilde{\lambda}^{\dot{1}} \\ \widetilde{\varrho}_B\tilde{\lambda}^{\dot{2}} \end{pmatrix} \\ &= \left(M^{AC}\widetilde{\varrho}_C\right)^* M^{BD}\widetilde{\varrho}_D\left(|\eta_1|^2 + |\eta_2|^2\right) + \widetilde{\varrho}_A^*\widetilde{\varrho}_B\left(\tilde{\lambda}^{\dot{1}}\lambda^1 + \tilde{\lambda}^{\dot{2}}\lambda^2\right)\end{aligned}$$

[8]The Dirac spinor $\psi_a^A = \overline{\psi}_A^{\dot{\alpha}} + \psi_\alpha^A$ is a shorthand for $\psi_a^A = \begin{pmatrix} 0 \\ \overline{\psi}_A^{\dot{\alpha}} \end{pmatrix} + \begin{pmatrix} \psi_\alpha^A \\ 0 \end{pmatrix} = \begin{pmatrix} \psi_\alpha^A \\ \overline{\psi}_A^{\dot{\alpha}} \end{pmatrix}$.

$$= 2E\left(\frac{(M^{AC}\widetilde{\varrho}_C)^*\, M^{BD}\widetilde{\varrho}_D}{4E^2} + \widetilde{\varrho}^*_A\widetilde{\varrho}_B\right) \tag{B.18}$$

by (B.11). To have $u_+^{\dagger A}u_+^B = 2E\delta^{AB}$, we have to set

$$\frac{(M^{AC}\widetilde{\varrho}_C)^*\, M^{BD}\widetilde{\varrho}_D}{4E^2} + \widetilde{\varrho}^*_A\widetilde{\varrho}_B = \delta^{AB}. \tag{B.19}$$

This in general is not possible. The failure to orthogonalize the + helicity part is due to the fact that the Hamiltonian is not Hermitian (CPT is violated). Nevertheless, for an arbitrarily given $\widetilde{\varrho}_A(E)$ and $\varrho_A(E)$, $u^A_{+a} = \widetilde{\varrho}_A\tilde{\lambda}^{\dot\alpha} + M^{AB}\widetilde{\varrho}_B\eta_\alpha$ can always be normalized and used for incoming states.

To summarize, the basis of normalized helicity states is:

$$u^A_{+a}(p) = \widetilde{\varrho}_A\tilde{\lambda}^{\dot\alpha} + M^{AB}\widetilde{\varrho}_B\eta_\alpha, \qquad u^A_{-a}(p) = \varrho^A\lambda_\alpha, \tag{B.20}$$

Following the recipe of field theory, in momentum space, we use $u_\pm(p)$ for the incoming fermion state functions with $\pm 1/2$ helicities.[9]

B.4 Outgoing functions

To find the outgoing states, we cannot just take the Dirac conjugate (i.e., $\bar{u}^{aA}_\pm$) of (B.20) as in ordinary field theory, because CPT is no longer invariant. Instead, we should restore CPT symmetry by adding the anti-chiral mass term $M_{AB} = (M^{AB})^*$ and get the new solution $u^{aA}_\pm$ and its Dirac conjugate $\bar{u}^{aA}_\pm$. In the end, we take $M_{AB} \to 0$ (formally keeping M^{AB} fixed) and $\bar{u}^{aA}_\pm$ in this limit will be the outgoing states for our theory with only a chiral mass term.

With both chiral and anti-chiral masses, the momentum is no longer lightlike. However, we can take the relativistic limit ($p \gg M$) and still decompose $p = \lambda\tilde{\lambda}$. At the order $\mathcal{O}(M)$, the helicity is still well-defined, and repeating the calculation above leads to

$$\begin{aligned} u^A_{+a}(p) &\approx \widetilde{\varrho}_A\tilde{\lambda}^{\dot\alpha} + M^{AB}\widetilde{\varrho}_B\eta_\alpha = \begin{pmatrix} M^{AB}\widetilde{\varrho}_B\eta_\alpha \\ \widetilde{\varrho}_A\tilde{\lambda}^{\dot\alpha}\end{pmatrix}, \\ u^A_{-a}(p) &\approx \varrho^A\lambda_\alpha + M_{AB}\varrho^B\tilde{\eta}^{\dot\alpha} = \begin{pmatrix} \varrho^A\lambda_\alpha \\ M_{AB}\varrho^B\tilde{\eta}^{\dot\alpha}\end{pmatrix}, \end{aligned} \tag{B.21}$$

where $\lambda^\alpha\eta_\alpha = \tilde{\lambda}_{\dot\alpha}\tilde{\eta}^{\dot\alpha} = 1$. This gives the Dirac conjugate:

$$\begin{aligned} \bar{u}^{aA}_+(p) &\approx \varrho^A\lambda^\alpha - M_{AB}\varrho^B\tilde{\eta}_{\dot\alpha} = \left(\varrho^A\lambda^\alpha,\ -M_{AB}\varrho^B\tilde{\eta}_{\dot\alpha}\right), \\ \bar{u}^{aA}_-(p) &\approx -M^{AB}\widetilde{\varrho}_B\eta^\alpha + \widetilde{\varrho}_A\tilde{\lambda}_{\dot\alpha} = \left(-M^{AB}\widetilde{\varrho}_B\eta^\alpha,\ \widetilde{\varrho}_A\tilde{\lambda}_{\dot\alpha}\right), \end{aligned} \tag{B.22}$$

[9] When studying the holomorphic structure of scattering amplitudes connected to twistor string theory, we relax the conditions (B.11), (B.17) and (B.19). Instead, we use the freedom (B.9) to set η_α to $\eta_\alpha = (1, 0)$ while scaling λ^α to $(1, Z = \lambda^2/\lambda^1)$.

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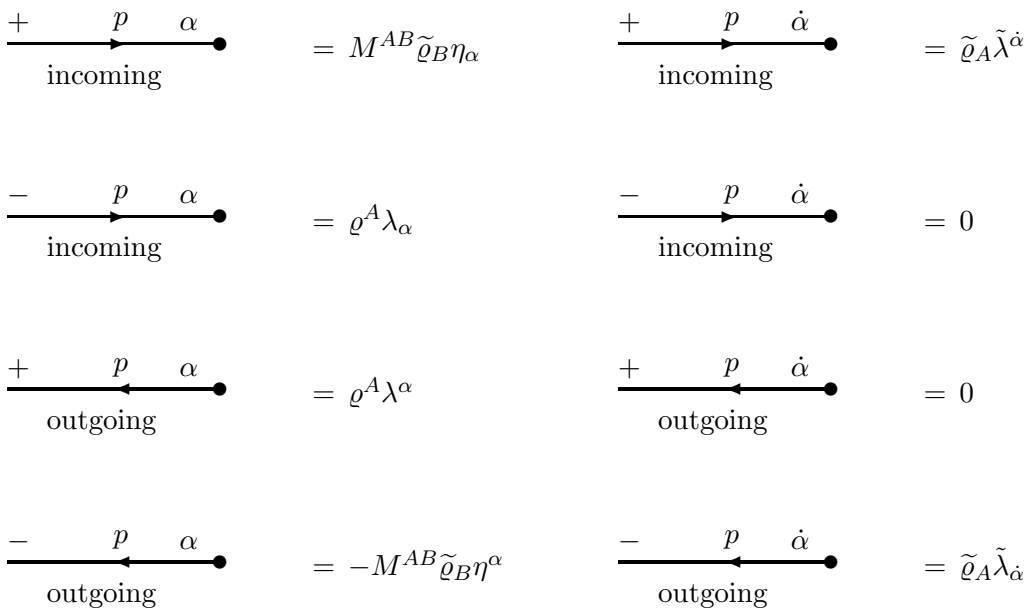

Figure 2: The Feynman rules for external fermions.

by the identities $M^{AB*} = M_{AB}$, $(\widetilde{\varrho}_A)^* = \varrho^A$ and $(\eta_\alpha)^* = -\tilde{\eta}_{\dot{\alpha}}$.[10] Setting $M_{AB} = 0$, we get the outgoing state functions:

$$\bar{u}_+^{aA}(p) = \varrho^A\lambda^\alpha, \qquad \bar{u}_-^{aA}(p) = -M^{AB}\widetilde{\varrho}_B\eta^\alpha + \widetilde{\varrho}_A\tilde{\lambda}_{\dot{\alpha}}. \tag{B.23}$$

Equivalently, (B.20) and (B.23) give the Feynman rules for external fermions depicted in figure 2.

B.5 Negative-frequency solutions

Similarly, we can solve for the negative-frequency solutions: $\psi(x) = v(p)e^{ip\cdot x}$. Repeating the calculation above, we find a basis of normalized helicity states for anti-fermions:[11]

$$v_+^A(p) = \widetilde{\varrho}_A\tilde{\lambda}^{\dot{\alpha}} - M^{AB}\widetilde{\varrho}_B\eta_\alpha, \qquad v_-^A(p) = \varrho^A\lambda_\alpha. \tag{B.24}$$

To compute $\bar{v}_\pm(p)$, we follow the method discussed after (B.23). For negative-frequency solutions, however, we have $(\lambda^\alpha)^* = -\tilde{\lambda}^{\dot{\alpha}}$ and accordingly we should choose $(\widetilde{\varrho}_A)^* = \varrho^A$ and $(\eta_\alpha)^* = \tilde{\eta}_{\dot{\alpha}}$ (contrary to the positive-frequency case). This leads to

$$\bar{v}_+^A(p) = -\varrho^A\lambda^\alpha, \qquad \bar{v}_-^A(p) = -\widetilde{\varrho}_A\tilde{\lambda}_{\dot{\alpha}} - M^{AB}\widetilde{\varrho}_B\eta^\alpha. \tag{B.25}$$

[10] In Minkowski signature, we have $(\lambda^\alpha)^* = \pm\tilde{\lambda}^{\dot{\alpha}}$ and we choose the + sign here for positive frequency solutions. In order to satisfy $\lambda^\alpha\eta_\alpha = \tilde{\lambda}_{\dot{\alpha}}\tilde{\eta}^{\dot{\alpha}} = 1$, we have to choose $(\eta_\alpha)^* = -\tilde{\eta}_{\dot{\alpha}}$ correspondingly with an extra minus sign.

[11] In fact, since CPT is violated, "anti-fermion" is not an appropriate term to describe the negative-frequency solution. Nevertheless, we use this name anyway for convenience.

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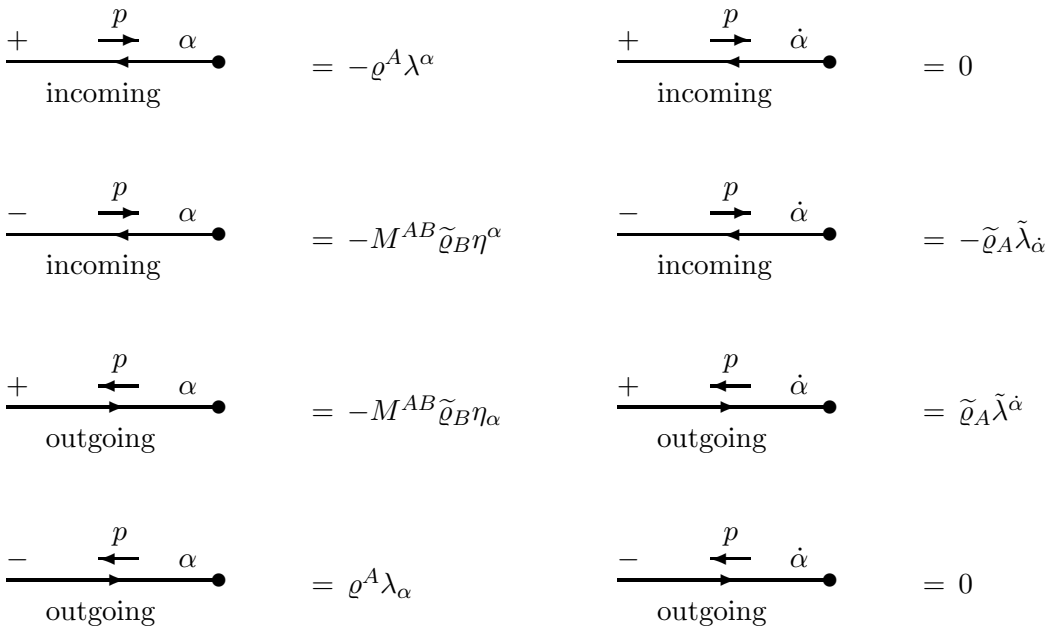

Figure 3: The Feynman rules for external anti-fermions.

In momentum space $\bar{v}_\pm(p)$ is used for the incoming anti-fermions with $\pm 1/2$ helicities and $v_\pm(p)$ for the outgoing anti-fermions with $\pm 1/2$ helicities. The corresponding Feynman rules for external anti-fermions are depicted in figure 3.

Notice that since $\overline{\psi}^a = (\psi^\alpha, \overline{\psi}_{\dot\alpha})$ is Majorana, the anti-fermions with adjoint color T_i and helicity $\pm$ are identical to the fermions with $\bar{T}_i$ and helicity $\mp$. We can treat any external fermionic legs as either "fermions" or "anti-fermions." The Feynman rules in figure 2 and figure 3 turn out to give the same resulting amplitude regardless of which way we choose, as long as all the directions of the arrows are consistent with the vertex rules depicted in figure 6.

B.6 Mass-deformed 3ϕ-interaction

We first study the case that M^{AB} is restricted to M^{ij} (i.e. $M^{A=4,B} = M^{B,A=4} = 0$). With the field identification (B.3), the mass-deformed 3ϕ-interaction term in (6.2) is given by

$$M^{ij}\epsilon_{jkl}\,\phi_i^*[\phi^k,\phi^l] = \frac{1}{4}\,M^{ij}\epsilon_{jkl}\,\phi_{4i}\,\epsilon^{kmn}\epsilon^{lpq}\,[\phi_{mn},\phi_{pq}] = \frac{1}{2}\,M^{ij}\epsilon^{klm}\,\phi_{4i}[\phi_{jk},\phi_{lm}]. \qquad \text{(B.26)}$$

On the other hand, with $M^{AB} \to M^{ij}$, the 3ϕ-interaction term in (B.5) reduces to

$$\begin{aligned} &\mathrm{tr}\left\{M^{AB}\epsilon^{CDEF}\,\phi_{AC}[\phi_{BD},\phi_{EF}]\right\} \\ &= \mathrm{tr}\left\{M^{ij}\epsilon^{4klm}\,\phi_{i4}[\phi_{jk},\phi_{lm}] + M^{ij}\epsilon^{k4lm}\,\phi_{ik}[\phi_{j4},\phi_{lm}] + \cdots\right\} \\ &= 2\,\mathrm{tr}\left\{M^{ij}\epsilon^{klm}\,\phi_{4i}[\phi_{jk},\phi_{lm}]\right\}. \end{aligned} \qquad \text{(B.27)}$$

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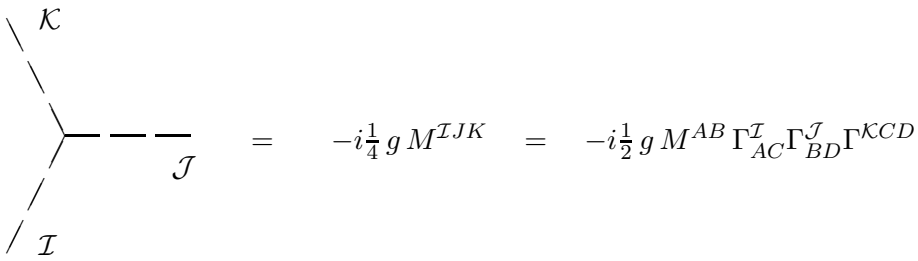

Figure 4: Feynman rules for 3-scalar vertices (planar part only and the algebraic factor for the color group ignored). Here, $\mathcal{I}$, $\mathcal{J}$ and $\mathcal{K}$ are $SU(4)$ R-symmetry indices in the representation **6**. $(\mathcal{I}, \mathcal{J}, \mathcal{K})$ are in the counterclockwise order on the page, since for the planar diagram we use the convention that the color group factor is the trace of adjoint matrices in counterclockwise order.

Comparing (B.26) with (B.27), we conclude that the 3ϕ-interaction term in (6.2) equals that in (B.5) when $M^{AB} \to M^{ij}$.

For general M^{AB}, we then have the 3ϕ-interaction in the Lagrangian:

$$\begin{aligned}
&\frac{g}{4}\,\mathrm{tr}\left\{M^{AB}\epsilon^{CDEF}\,\phi_{AC}[\phi_{BD},\phi_{EF}]\right\} = \frac{g}{4}\,\mathrm{tr}\left\{M^{AB}\epsilon^{CDEF}\,\phi_{BD}[\phi_{EF},\phi_{AC}]\right\} = \cdots \\
&= \frac{g}{2}\,M^{AB}\Gamma^{\mathcal{I}}_{AC}\Gamma^{\mathcal{J}}_{BD}\Gamma^{\mathcal{K}CD}\,\mathrm{tr}\left\{\phi_{\mathcal{I}}\left[\phi_{\mathcal{J}},\phi_{\mathcal{K}}\right]\right\} = \frac{g}{2}\,M^{AB}\Gamma^{\mathcal{K}}_{AC}\Gamma^{\mathcal{I}}_{BD}\Gamma^{\mathcal{J}CD}\,\mathrm{tr}\left\{\phi_{\mathcal{I}}\left[\phi_{\mathcal{J}},\phi_{\mathcal{K}}\right]\right\} = \cdots \\
&= \frac{g}{4}\,M^{AB}\Gamma^{[\mathcal{I}}_{AC}\Gamma^{\mathcal{J}}_{BD}\Gamma^{\mathcal{K}]}_{EF}\epsilon^{CDEF}\,\mathrm{tr}\left\{\phi_{\mathcal{I}}\left[\phi_{\mathcal{J}},\phi_{\mathcal{K}}\right]\right\} = \frac{g}{4}\,M^{\mathcal{I}JK}\,\mathrm{tr}\left\{\phi_{\mathcal{I}}\left[\phi_{\mathcal{J}},\phi_{\mathcal{K}}\right]\right\}
\end{aligned} \tag{B.28}$$

where "$\cdots$" means cyclic permutation of indices. The planar part of the corresponding Feynman rule (3-scalar vertex) is depicted in figure 4.

B.7 Other Feynman rules

The Feynman rules involving the gluons do not change with the (anti-)chiral mass term. For our purpose, instead of arbitrary ϵ_μ, we use helicity to describe the polarization. To get a positive (negative) helicity polarization vector, we set

$$\begin{aligned}
\epsilon_{\mu,+} \to \tilde{\epsilon}_{\alpha\dot{\alpha}} &= \frac{\xi_\alpha \tilde{\lambda}_{\dot{\alpha}}}{\langle \xi, \lambda\rangle}, \qquad \epsilon_{\mu,-} \to \epsilon_{\alpha\dot{\alpha}} = \frac{\lambda_\alpha \tilde{\xi}_{\dot{\alpha}}}{[\tilde{\lambda}, \tilde{\xi}]}, \\
\epsilon^*_{\mu,+} \to \epsilon_{\alpha\dot{\alpha}} &= \frac{\lambda_\alpha \tilde{\xi}_{\dot{\alpha}}}{[\tilde{\lambda}, \tilde{\xi}]}, \qquad \epsilon^*_{\mu,-} \to \tilde{\epsilon}_{\alpha\dot{\alpha}} = \frac{\xi_\alpha \tilde{\lambda}_{\dot{\alpha}}}{\langle \xi, \lambda\rangle},
\end{aligned} \tag{B.29}$$

where $\xi(\tilde{\xi})$ is arbitrary but not a multiple of $\lambda(\tilde{\lambda})$ (See [3]). Feynman rules for external gluons are shown in figure 5.

All other Feynman rules are exactly the same as those in the massless theory. In particular, we list the fermion-gluon vertices in figure 6, fermion-scalar vertices in figure 7 and scalar-gluon vertex in figure 8.

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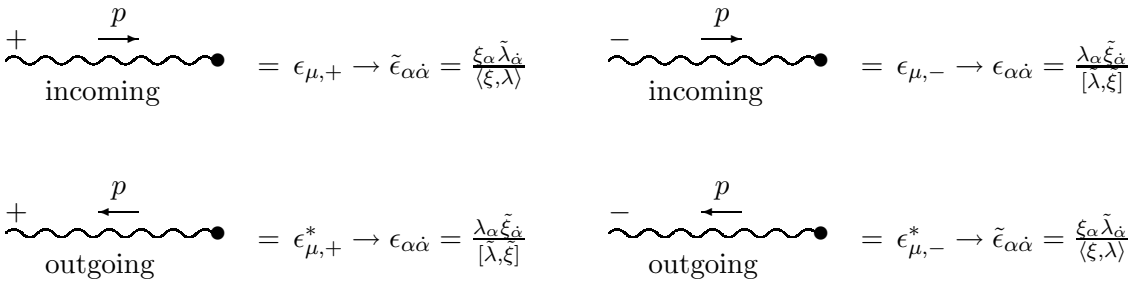


Figure 5: The Feynman rules for external gluons.

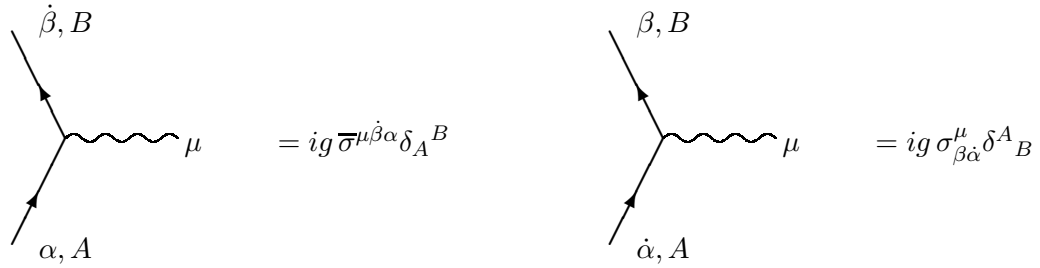


Figure 6: Feynman rules for fermion-gluon vertices. Here, A and B are $SU(4)$ R-symmetry indices in the **4** or $\bar{\mathbf{4}}$ representation. The algebraic factor for the color group is ignored.

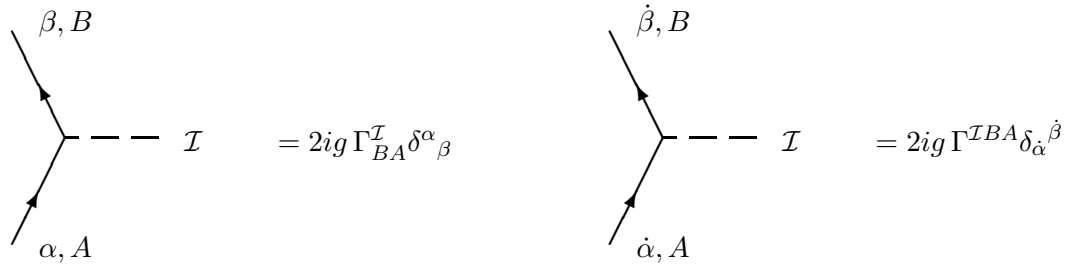
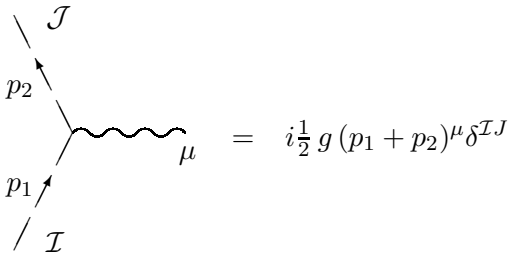


Figure 7: Feynman rules for fermion-scalar vertices. Here, A and B are $SU(4)$ R-symmetry indices in the **4** or $\bar{\mathbf{4}}$ representation and $\mathcal{I}$ is the index in **6** representation. The algebraic factor for the color group is ignored.

C. Detailed computation for Feynman diagrams

In this section, we present the calculation of the tree-level planar Feynman diagrams in detail for the scattering amplitudes presented in section 3. Some techniques used here can be found in [46].



$$= \quad i\tfrac{1}{2}\, g\,(p_1+p_2)^{\mu}\delta^{\mathcal{I}J}$$

Figure 8: Feynman rule for the scalar-gluon vertex. Here, $\mathcal{I}$ and $\mathcal{J}$ are $SU(4)$ R-symmetry indices in the **6** representation; p_1 and p_2 represent the physical momenta if the corresponding dashed line happens to be an external leg. The algebraic factor for the color group is ignored.

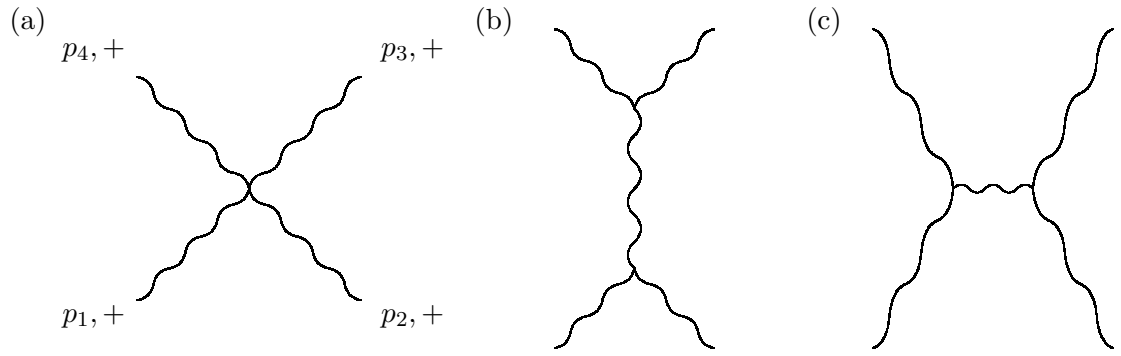


Figure 9: Planar Feynman diagrams that contribute to the MHV amplitude $A_{\mathcal{O}(M^0)}(+1,+1,-1,-1)$. In order to directly apply the Feynman rules as in appendix B, in the figures we are not using the convention that all external legs are incoming (instead, all depicted momenta and helicities are physical).

C.1 MHV amplitudes (extended MHV at $\mathcal{O}(M^0)$)

In this subsection, we calculate MHV diagrams without mass contribution.

C.1.1 $A_{\mathcal{O}(M^0)}(+1,+1,-1,-1)$

This is a 4-gluon scattering amplitude. The Feynman diagrams are shown in figure 9. Accordingly, the amplitude of 4 gluons are the same as that without the mass term, which is given by (See, e.g., [9])

$$A_{\mathcal{O}(M^0)}(+1,+1,-1,-1) = \frac{ig^2}{2}\frac{\langle 3,4\rangle^4}{\prod_{i=1}^{4}\langle i,i+1\rangle}. \tag{C.1}$$

C.1.2 $A_{\mathcal{O}(M^0)}(+1/2,+1,-1,-1/2)$

This is a 2-gluon and 2-fermion scattering amplitude. The Feynman rules give two contributions listed in figure 10:

$$A_a = \epsilon^{*}_{3\,\nu+}\varrho_4^B\lambda_4^{\beta}\left(ig\sigma^{\nu}_{\beta\dot\beta}\right)\left[\frac{i(p_1+p_2)^{\dot\beta\alpha}\delta^{A}{}_{B}}{(p_1+p_2)^2}\right]\left(ig\sigma^{\mu}_{\alpha\dot\alpha}\right)\widetilde{\varrho}_{1A}\tilde\lambda_1^{\dot\alpha}\,\epsilon_{2\,\mu+}. \tag{C.2}$$

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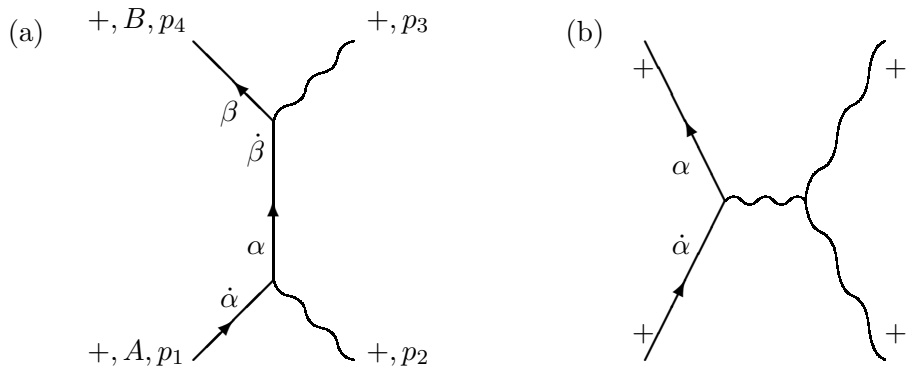

Figure 10: Planar Feynman diagrams for the MHV amplitude $A_{\mathcal{O}(M^0)}(+1/2,+1,-1,-1/2)$. Wavy lines are gluons and solid lines are fermions. Time is in the vertical upward direction.

Here, $\epsilon_{2\,\nu+}$ and $\epsilon^*_{3\,\nu+}$ are gluon polarization vectors for the particles with momenta p_2 and p_3, respectively. $\sigma^\nu_{\beta\dot\beta}$ are Pauli matrices. By the rules in (B.29) and in figure 5, we have

$$\epsilon_{\mu+}(p_2)\sigma^\mu_{\alpha\dot\alpha} = \frac{\xi_{2\alpha}\tilde\lambda_{2\dot\alpha}}{\langle\xi_2,2\rangle}, \qquad \epsilon^*_{\nu+}(p_3)\sigma^\nu_{\beta\dot\beta} = \frac{\lambda_{3\beta}\tilde\xi_{3\dot\beta}}{[3,\tilde\xi_3]}, \tag{C.3}$$

where ξ_2 and ξ_3 are arbitrary spinors.

$$\begin{aligned} A_a &= \frac{ig^2\varrho_4^A\widetilde\varrho_{1A}}{\langle1,2\rangle[1,2][3,\tilde\xi_3]\langle\xi_2,\lambda_2\rangle}\ \lambda_4^\beta\lambda_{3\beta}\tilde\xi_{3\dot\beta}\left(\tilde\lambda_1^{\dot\beta}\lambda_1^\alpha+\tilde\lambda_2^{\dot\beta}\lambda_2^\alpha\right)\xi_{2\alpha}\tilde\lambda_{2\dot\alpha}\tilde\lambda_1^{\dot\alpha} \\ &= \frac{ig^2\varrho_4^A\widetilde\varrho_{1A}}{\langle1,2\rangle[3,\tilde\xi_3]\langle\xi_2,\lambda_2\rangle}\ \langle3,4\rangle\left([\tilde\xi_3,1]\langle1,\xi_2\rangle+[\tilde\xi_3,2]\langle2,\xi_2\rangle\right). \end{aligned} \tag{C.4}$$

Gauge-fixing the external gluon polarizations by taking $\xi_2=\lambda_3$ and $\tilde\xi_3=\tilde\lambda_2$, we get[12]

$$A_a \to ig^2\varrho_4^A\widetilde\varrho_{1A}\ \frac{\langle3,4\rangle\langle1,3\rangle[2,1]}{\langle1,2\rangle\langle2,3\rangle[2,3]} = -ig^2\varrho_4^A\widetilde\varrho_{1A}\ \frac{\langle3,4\rangle^3\langle1,3\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}, \tag{C.5}$$

where we have used the identity

$$\frac{[2,1]}{[2,3]} = \frac{\langle4,3\rangle}{\langle4,1\rangle}, \tag{C.6}$$

which follows from momentum conservation, $\sum_{i=1}^2\lambda_i^\alpha\tilde\lambda_i^{\dot\alpha}=\sum_{i=3}^4\lambda_i^\alpha\tilde\lambda_i^{\dot\alpha}$.

The diagram of figure 10b gives

$$\begin{aligned} A_b &= \epsilon^*_{3\,\nu+}(ig)\left[\eta^{\mu\nu}(p_2+p_3)^\varrho+\eta^{\nu\varrho}(p_2-2p_3)^\mu+\eta^{\rho\mu}(p_3-2p_2)^\nu\right]\epsilon_{2\,\mu+} \\ &\quad\times\left[i\frac{\eta_{\sigma\rho}}{(p_2-p_3)^2}\right]\varrho_4^B\lambda_4^\alpha\left(ig\sigma^\sigma_{\alpha\dot\alpha}\delta^A{}_B\right)\widetilde\varrho_{1A}\tilde\lambda_1^{\dot\alpha}, \end{aligned} \tag{C.7}$$

where $p_2^\mu\epsilon_{2\mu+}=\epsilon^*_{3\nu+}p_3^\nu=0$ and $\eta^{\mu\nu}\epsilon^*_{3\nu+}\epsilon_{2\mu+}$ can be expressed again in the form of figure 5 with the help of the identity (A.9).

It follows that

$$A_b = \frac{-ig^2\varrho_4^A\widetilde\varrho_{1A}}{2\langle2,3\rangle[2,3][3,\tilde\xi_3]\langle\xi_2,2\rangle}\ \lambda_4^\alpha\tilde\lambda_1^{\dot\alpha}$$

[12]Henceforth, an arrow $\to$ represents a particular gauge choice.

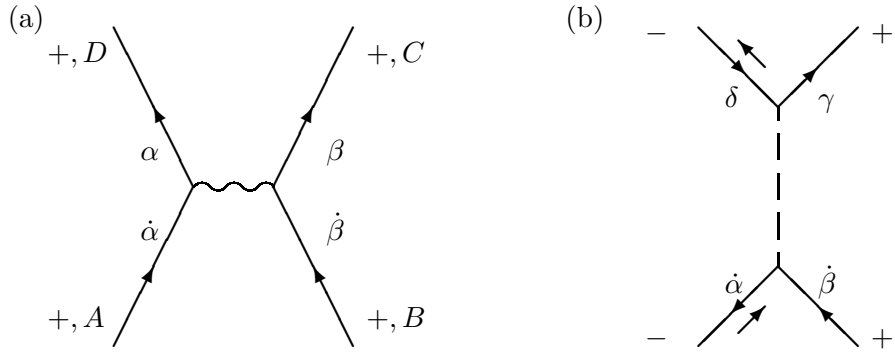

Figure 11: Planar Feynman diagrams for $A_{\mathcal{O}(M^0)}(+1/2,+1/2,-1/2,-1/2)$. In (b), the helicities for the 1st and 4th particles are flipped since we treat them as anti-fermions.

$$
\begin{aligned}
&\times \left\{\tilde{\xi}_3^{\dot\beta}\lambda_3^{\beta}\tilde{\lambda}_{2\dot\beta}\,\xi_{2\beta}(p_2+p_3)_{\alpha\dot\alpha} - 2\tilde{\xi}_{3\dot\alpha}\lambda_{3\alpha}\tilde{\lambda}_{2\dot\beta}\,\xi_{2\beta}p_3^{\dot\beta\beta} - 2\tilde{\xi}_3^{\dot\beta}\lambda_3^{\beta}\,\xi_{2\alpha}\tilde{\lambda}_{2\dot\alpha}p_{2\dot\beta\beta}\right\} \\
&= \frac{-ig^2\varrho_4^A\widetilde{\varrho}_{1A}}{2\langle 2,3\rangle[2,3][3,\tilde{\xi}_3]\langle\xi_2,2\rangle}\Big\{\langle 4,2\rangle[2,1]\langle 3,\xi_2\rangle[2,\tilde{\xi}_3] + \langle 4,3\rangle[3,1]\langle 3,\xi_2\rangle[2,\tilde{\xi}_3] \\
&\qquad -2\langle 4,3\rangle[\tilde{\xi}_3,1][2,3]\langle 3,\xi_2\rangle - 2\langle 4,\xi_2\rangle\langle 3,2\rangle[2,1][2,\tilde{\xi}_3]\Big\} \\
&\to 0
\end{aligned}
\tag{C.8}
$$

in the gauge $\xi_2 = \lambda_3$ and $\tilde{\xi}_3 = \tilde{\lambda}_2$.

Therefore,

$$
A_{\mathcal{O}(M^0)}(+1/2,+1,-1,-1/2) = A_a + A_b = ig^2\varrho_4^A\widetilde{\varrho}_{1A}\,\frac{\langle 3,4\rangle^3\langle 1,3\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}. \tag{C.9}
$$

C.1.3 $A_{\mathcal{O}(M^0)}(+1/2,+1/2,-1/2,-1/2)$

This is a 4-fermion scattering amplitude. The Feynman diagram in figure 11(a) gives

$$
\begin{aligned}
A_a &= (\varrho_4^D\lambda_4^{\alpha})(\widetilde{\varrho}_{1A}\tilde{\lambda}_1^{\dot\alpha})\left(ig\sigma^{\mu}_{\alpha\dot\alpha}\delta^A{}_D\right)\left[\frac{-i\eta_{\mu\nu}}{(p_1-p_4)^2}\right]\left(ig\sigma^{\nu}_{\beta\dot\beta}\delta^B{}_C\right)(\varrho_3^C\lambda_3^{\beta})(\widetilde{\varrho}_{2B}\tilde{\lambda}_2^{\dot\beta}) \\
&= 2ig^2\varrho_4^A\widetilde{\varrho}_{1A}\varrho_3^B\widetilde{\varrho}_{2B}\frac{\langle 3,4\rangle[1,2]}{\langle 1,4\rangle[1,4]} = 2ig^2\varrho_4^A\widetilde{\varrho}_{1A}\varrho_3^B\widetilde{\varrho}_{2B}\frac{\langle 3,4\rangle^3\langle 1,2\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}
\end{aligned}
\tag{C.10}
$$

by the identity (A.6).

Since the anti-fermions with adjoint color T_i and helicity $\pm$ are identical to the fermions with $\bar{T}_i$ and helicity $\mp$, we should consider the s-channel as shown on figure 11(b), which gives

$$
\begin{aligned}
A_b &= (\varrho_4^D\lambda_{4\delta})(\varrho_3^C\lambda_3^{\gamma})\left(2ig\Gamma^{\mathcal{J}}_{CD}\delta^{\delta}{}_{\gamma}\right)\left[\frac{-i\delta_{\mathcal{I}\mathcal{J}}}{(p_1+p_2)^2}\right]\left(2ig\Gamma^{\mathcal{I}AB}\delta_{\dot\beta}{}^{\dot\alpha}\right)(-\widetilde{\varrho}_{1A}\tilde{\lambda}_{1\dot\alpha})(\widetilde{\varrho}_{2B}\tilde{\lambda}_2^{\dot\beta}) \\
&= -2ig^2(\varrho_4^A\widetilde{\varrho}_{1A}\varrho_3^B\widetilde{\varrho}_{2B} - \varrho_3^A\widetilde{\varrho}_{1A}\varrho_4^B\widetilde{\varrho}_{2B})\frac{\langle 3,4\rangle}{\langle 1,2\rangle} \\
&= -2ig^2(\varrho_4^A\widetilde{\varrho}_{1A}\varrho_3^B\widetilde{\varrho}_{2B} - \varrho_3^A\widetilde{\varrho}_{1A}\varrho_4^B\widetilde{\varrho}_{2B})\frac{\langle 3,4\rangle^2\langle 2,3\rangle\langle 4,1\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}
\end{aligned}
\tag{C.11}
$$

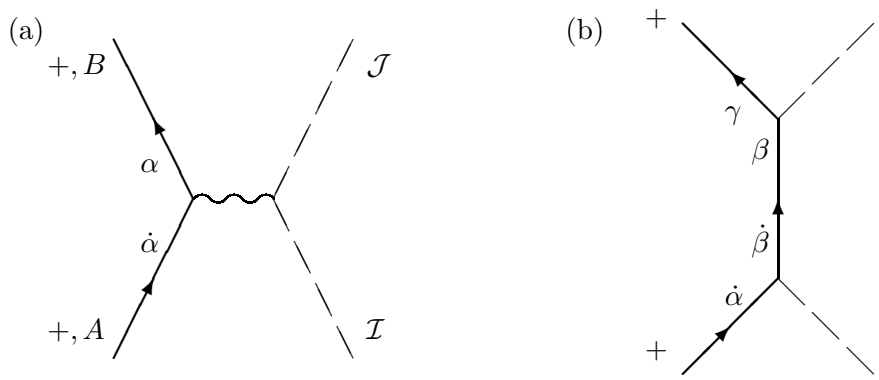

Figure 12: Planar Feynman diagram for $A_{\mathcal{O}(M^0)}(+1/2, 0, 0, -1/2)$. The dashed lines are scalars.

by the identity (A.13).

Thus,

$$\begin{aligned} & A_{\mathcal{O}(M^0)}(+1/2,+1/2,-1/2,-1/2) = A_a + A_b \\ = & -\frac{2ig^2\langle 3,4\rangle^2}{\prod_{i=1}^4\langle i,i+1\rangle}\left\{\varrho_4^A\widetilde{\varrho}_{1A}\varrho_3^B\widetilde{\varrho}_{2B}\left(\langle 2,3\rangle\langle 4,1\rangle - \langle 3,4\rangle\langle 1,2\rangle\right) - \varrho_3^A\widetilde{\varrho}_{1A}\varrho_4^B\widetilde{\varrho}_{2B}\langle 2,3\rangle\langle 4,1\rangle\right\} \\ \longrightarrow & \frac{2ig^2\langle 3,4\rangle^2}{\prod_{i=1}^4\langle i,i+1\rangle}\left\{\varrho_4^A\widetilde{\varrho}_{1A}\widetilde{\varrho}_{2B}\varrho_3^B\langle 1,3\rangle\langle 2,4\rangle + \varrho_3^A\widetilde{\varrho}_{1A}\widetilde{\varrho}_{2B}\varrho_4^B\langle 2,3\rangle\langle 4,1\rangle\right\}, \end{aligned} \tag{C.12}$$

where in the last line we scale $(\lambda_i^1, \lambda_i^2) = (1, Z_i)$ and thus $\langle i,j\rangle = Z_j - Z_i$.

C.1.4 $A_{\mathcal{O}(M^0)}(+1/2, 0, 0, -1/2)$

The Feynman diagram in figure 12(a) gives:

$$\begin{aligned} A_a &= (\varrho_4^B\lambda_4^\alpha)(\widetilde{\varrho}_{1A}\tilde{\lambda}_1^{\dot\alpha})\left(ig\,\sigma^\mu_{\alpha\dot\alpha}\delta^A{}_B\right)\left[\frac{-i\eta_{\mu\nu}}{(p_1-p_4)^2}\right]\left(\frac{ig}{2}(p_2+p_3)^\nu\delta^{\mathcal{I}\mathcal{J}}\right)\varphi_{2\mathcal{I}}\varphi_{3\mathcal{J}} \\ &= -ig^2\varrho_4^A\widetilde{\varrho}_{1A}\varphi_2^{\mathcal{I}}\varphi_{3\mathcal{I}}\frac{\langle 4,2\rangle[1,2]+\langle 4,3\rangle[1,3]}{2\langle 1,4\rangle[1,4]} \\ &= ig^2\varrho_4^A\widetilde{\varrho}_{1A}\varphi_2^{BC}\varphi_{3BC}\frac{\langle 1,2\rangle\langle 2,4\rangle\langle 3,4\rangle^2}{\prod_{i=1}^4\langle i,i+1\rangle}, \end{aligned} \tag{C.13}$$

where $\varphi_{3\mathcal{J}}$ and $\varphi_{2\mathcal{I}}$ are used for the external scalar particles and (A.16) is used.
figure 12(b) gives:

$$\begin{aligned} A_b &= \varphi_{3\mathcal{J}}\varrho_4^B\lambda_4^\gamma\left(2ig\Gamma^{\mathcal{J}}_{BD}\delta^\beta{}_\gamma\right)\left[\frac{i(p_1+p_2)_{\beta\dot\beta}\delta_C{}^D}{(p_1+p_2)^2}\right]\left(2ig\Gamma^{\mathcal{I}CA}\delta_{\dot\alpha}{}^{\dot\beta}\right)\widetilde{\varrho}_{1A}\tilde{\lambda}_1^{\dot\alpha}\,\varphi_{2\mathcal{I}} \\ &= 2ig^2\varrho_4^B\widetilde{\varrho}_{1A}\varphi_2^{CA}\varphi_{3BC}\frac{\langle 4,1\rangle[1,1]+\langle 4,2\rangle[2,1]}{\langle 1,2\rangle[1,2]} \\ &= 2ig^2\varrho_4^B\widetilde{\varrho}_{1A}\varphi_2^{CA}\varphi_{3BC}\frac{\langle 2,3\rangle\langle 3,4\rangle\langle 4,1\rangle\langle 2,4\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}. \end{aligned} \tag{C.14}$$

Altogether,

$$A_{\mathcal{O}(M^0)}(+1/2, 0, 0, -1/2) = A_a + A_b$$

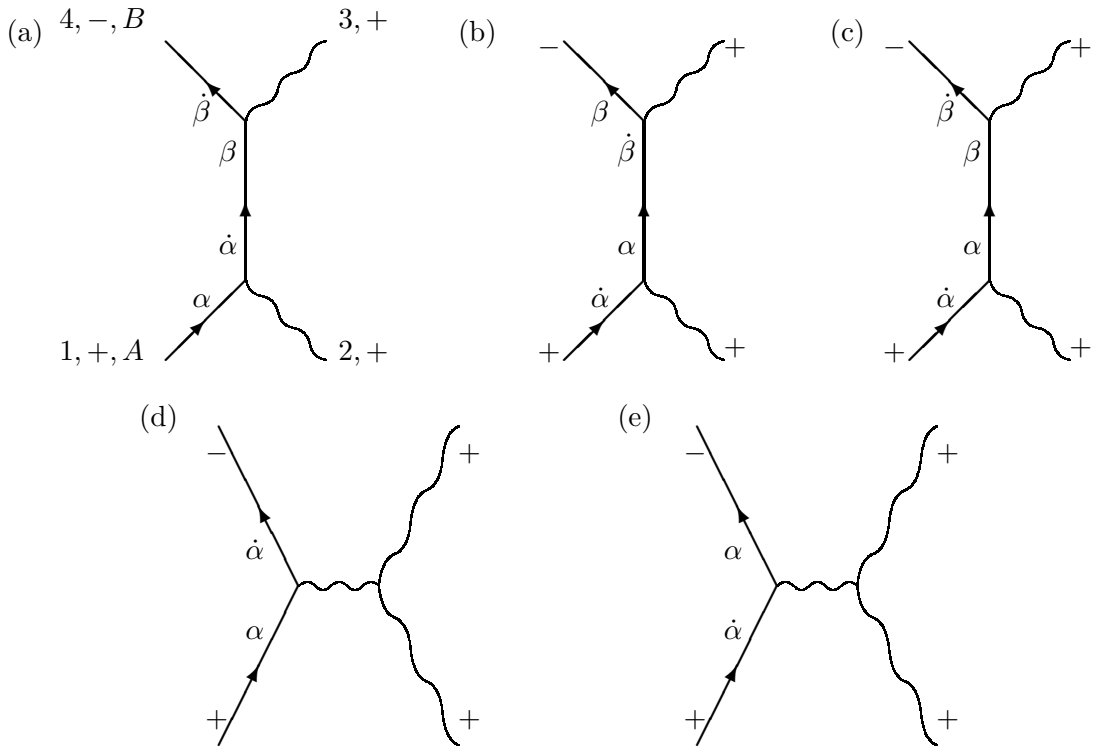

Figure 13: Planar Feynman diagrams with two external fermions and two external gluons corresponding to the extended MHV amplitude $A_{\mathcal{O}(M)}(+1/2,+1,-1,+1/2)$.

$$= \frac{2ig^2\langle 3,4\rangle\langle 2,4\rangle}{\prod_{i=1}^{4}\langle i,i+1\rangle}\Big\{\frac{1}{2}\varrho_4^A\widetilde{\varrho}_{1A}\varphi_2^{BC}\varphi_{3BC}\langle 1,2\rangle\langle 3,4\rangle + \varrho_4^B\widetilde{\varrho}_{1A}\varphi_2^{CA}\varphi_{3BC}\langle 2,3\rangle\langle 4,1\rangle\Big\}. \tag{C.15}$$

C.2 Extended MHV amplitudes at $\mathcal{O}(M)$

In this subsection we will calculate the extended MHV diagrams with the contribution of the mass M^{AB} up to the first order.

C.2.1 $A_{\mathcal{O}(M)}(+1/2,+1,-1,+1/2)$

The Feynman diagrams are listed in figure 13:

$$\begin{aligned}
A_a &= \epsilon^*_{3\nu+}\widetilde{\varrho}_{4B}\tilde{\lambda}_{4\dot{\beta}}\left(ig\overline{\sigma}^{\nu\dot{\beta}\beta}\right)\left[\frac{i(p_1+p_2)_{\beta\dot{\alpha}}\,\delta_A{}^B}{(p_1+p_2)^2}\right]\left(ig\overline{\sigma}^{\mu\dot{\alpha}\alpha}\right)M^{AC}\widetilde{\varrho}_{1C}\eta_{1\alpha}\epsilon_{2\mu+}\\
&= \frac{ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}}{\langle 1,2\rangle[1,2][3,\tilde{\xi}_3]\langle\xi_2,2\rangle}\,\tilde{\lambda}_{4\dot{\beta}}\,\tilde{\xi}_3^{\dot{\beta}}\lambda_3^{\beta}\left[\lambda_{1\beta}\tilde{\lambda}_{1\dot{\alpha}}+\lambda_{2\beta}\tilde{\lambda}_{2\dot{\alpha}}\right]\,\tilde{\lambda}_2^{\dot{\alpha}}\xi_2^{\alpha}\eta_{1\alpha}\\
&= ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}\frac{[4,\tilde{\xi}_3]\langle 3,1\rangle\langle\xi_2,\eta_1\rangle}{\langle 1,2\rangle[3,\tilde{\xi}_3]\langle\xi_2,2\rangle}\to ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}\frac{[4,2]\langle 3,1\rangle\langle 3,\eta_1\rangle}{\langle 1,2\rangle[3,2]\langle 3,2\rangle}\\
&= ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}\frac{\langle 3,1\rangle\langle 3,4\rangle\{\langle 3,1\rangle\langle 3,\eta_1\rangle\}}{\prod_{i=1}^{4}\langle i,i+1\rangle},
\end{aligned} \tag{C.16}$$

in the gauge $\xi_2=\lambda_2$ and $\tilde{\xi}_3=\tilde{\lambda}_3$.

Similarly, we have

$$\begin{aligned}
A_b &= \epsilon^*_{3\nu+}(-M^{BC}\widetilde{\varrho}_{4C}\eta_4^\beta)\left(ig\sigma^\nu_{\beta\dot\beta}\right)\left[\frac{i(p_1+p_2)^{\dot\beta\alpha}\,\delta^A{}_B}{(p_1+p_2)^2}\right]\left(ig\sigma^\mu_{\alpha\dot\alpha}\right)\rho\tilde{}_{1A}\tilde\lambda_1^{\dot\alpha}\epsilon_{2\mu+}\\
&= -\frac{ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}}{\langle1,2\rangle[1,2][3,\tilde\xi_3]\langle\xi_2,2\rangle}\,\eta_4^\beta\lambda_{3\beta}\,\tilde\xi_{3\dot\beta}\left[\tilde\lambda_1^{\dot\beta}\lambda_1^\alpha+\tilde\lambda_2^{\dot\beta}\lambda_2^\alpha\right]\,\xi_{2\alpha}\tilde\lambda_{2\dot\alpha}\tilde\lambda_1^{\dot\alpha}\\
&= -ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}\frac{\langle3,\eta_4\rangle[\tilde\xi_3,1]\langle1,\xi_2\rangle}{\langle1,2\rangle[3,\tilde\xi_3]\langle\xi_2,2\rangle}\\
&\to -ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}\frac{\langle3,\eta_4\rangle[1,2]\langle1,3\rangle}{\langle1,2\rangle[3,2]\langle2,3\rangle}\\
&= -ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}\frac{\langle3,1\rangle\langle3,4\rangle\left\{\langle3,4\rangle\langle3,\eta_4\rangle\right\}}{\prod_{i=1}^4\langle i,i+1\rangle}
\end{aligned}\tag{C.17}$$

in the same gauge.

Meanwhile,

$$\begin{aligned}
A_c &= \epsilon^*_{3\,\nu+}\widetilde{\varrho}_{4B}\tilde\lambda_{4\dot\beta}\left(ig\overline{\sigma}^{\nu\dot\beta\beta}\right)\left[i\frac{M^{AB}\delta_\beta{}^\alpha}{(p_1+p_2)^2}\right](ig\sigma_{\mu\alpha\dot\alpha})\,\widetilde{\varrho}_{1A}\tilde\lambda_1^{\dot\alpha}\,\epsilon_{2\,\mu+}\\
&= \frac{ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}}{\langle1,2\rangle]1,2][3,\tilde\xi_3]\langle\xi_2,2\rangle}\,\tilde\lambda_{4\dot\beta}\,\tilde\xi_3^{\dot\beta}\lambda_3^\alpha\,\xi_{2\alpha}\tilde\lambda_{2\dot\alpha}\tilde\lambda_1^{\dot\alpha}\to 0,
\end{aligned}\tag{C.18}$$

for $\lambda_3^\alpha\xi_{2\alpha}\to\langle3,3\rangle=0$.

Furthermore, since the 3-gluon vertices in diagrams (d) and (e) have exactly the same structure as that in figure 10(b), we have the same vanishing result as (C.8):

$$A_d\to 0,\qquad A_e\to 0,\tag{C.19}$$

as we are taking the same gauge, $\xi_2=\lambda_2$ and $\tilde\xi_3=\tilde\lambda_3$.

As a result,

$$\begin{aligned}
&A_{\mathcal{O}(M)}(+1/2,+1,-1,+1/2)=A_a+A_b+A_c+A_d+A_e\\
=\;&\frac{ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}}{2}\,\frac{\langle3,1\rangle\langle3,4\rangle\left\{\langle3,1\rangle\langle3,\eta_1\rangle-\langle3,4\rangle\langle3,\eta_4\rangle\right\}}{\prod_{i=1}^4\langle i,i+1\rangle}\\
\longrightarrow\;&\frac{ig^2M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}}{2}\,\frac{\langle3,1\rangle\langle3,4\rangle\langle4,1\rangle}{\prod_{i=1}^4\langle i,i+1\rangle},
\end{aligned}\tag{C.20}$$

where in the last line we scale $(\lambda_i^1,\lambda_i^2)=(1,Z_i)$ and $(\eta_i^1,\eta_i^2)=(0,1)$; accordingly $\langle i,j\rangle=-\langle j,i\rangle=Z_j-Z_i$ and $\langle i,\eta_j\rangle=-\langle\eta_j,i\rangle=1$.[13]

C.2.2 $A_{\mathcal{O}(M)}(+1/2,+1/2,-1/2,+1/2)$

The Feynman diagrams are listed in figure 14. Figures (a)–(d) are diagrams exchanging a gluon propagator while (e) and (f) exchange a scalar propagator.

[13]Henceforth, a long arrow $\longrightarrow$ represents the scaling $(\lambda_i^1,\lambda_i^2)=(1,Z_i)$ and $(\eta_i^1,\eta_i^2)=(0,1)$.

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The Feynman rules give us

$$
\begin{aligned}
A_a &= (-M^{DE}\widetilde{\varrho}_{4E}\eta_4^{\delta})(\widetilde{\varrho}_{1A}\tilde{\lambda}_1^{\dot{\alpha}})\left(ig\sigma^{\mu}_{\delta\dot{\alpha}}\delta^{A}{}_{D}\right)\left[\frac{-i\eta_{\mu\nu}}{(p_1-p_4)^2}\right]\left(ig\sigma^{\nu}_{\gamma\dot{\beta}}\delta^{B}{}_{C}\right)(\varrho_3^C\lambda_3^{\gamma})(\widetilde{\varrho}_{2B}\tilde{\lambda}_2^{\dot{\beta}}) \\
&= -2ig^2(M^{AD}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4D})(\varrho_3^C\widetilde{\varrho}_{2C})\frac{\langle 3,\eta_4\rangle[1,2]}{\langle 1,4\rangle[1,4]} \\
&= -2ig^2(M^{AD}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4D})(\varrho_3^C\widetilde{\varrho}_{2C})\frac{\langle 1,2\rangle\langle 3,4\rangle\left\{\langle 3,4\rangle\langle 3,\eta_4\rangle\right\}}{\prod_{i=1}^{4}\langle i,i+1\rangle}
\end{aligned}
\tag{C.21}
$$

by the identity (A.6).

$$
\begin{aligned}
A_b &= (\widetilde{\varrho}_{4D}\tilde{\lambda}_{4\dot{\delta}})(M^{AE}\widetilde{\varrho}_{1E}\eta_{1\alpha})\left(ig\overline{\sigma}^{\mu\dot{\delta}\alpha}\delta_{A}{}^{D}\right)\left[\frac{-i\eta_{\mu\nu}}{(p_1-p_4)^2}\right]\left(ig\sigma^{\nu}_{\gamma\dot{\beta}}\delta^{B}{}_{C}\right)(\varrho_3^C\lambda_3^{\gamma})(\widetilde{\varrho}_{2B}\tilde{\lambda}_2^{\dot{\beta}}) \\
&= -2ig^2(M^{AD}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4D})(\varrho_3^C\widetilde{\varrho}_{2C})\frac{\langle 3,\eta_1\rangle[2,4]}{\langle 1,4\rangle[1,4]} \\
&= 2ig^2(M^{AD}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4D})(\varrho_3^C\widetilde{\varrho}_{2C})\frac{\langle 1,2\rangle\langle 3,4\rangle\left\{\langle 3,1\rangle\langle 3,\eta_1\rangle\right\}}{\prod_{i=1}^{4}\langle i,i+1\rangle}
\end{aligned}
\tag{C.22}
$$

by the identity (A.6).

$$
\begin{aligned}
A_c &= (\widetilde{\varrho}_{4D}\tilde{\lambda}_4^{\dot{\delta}})(\varrho_3^C\lambda_3^{\gamma})\left(ig\sigma^{\mu}_{\gamma\dot{\delta}}\delta^{D}{}_{C}\right)\left[\frac{-i\eta_{\mu\nu}}{(p_1+p_2)^2}\right]\left(ig\overline{\sigma}^{\nu\dot{\alpha}\beta}\delta_{B}{}^{A}\right)(-\widetilde{\varrho}_{1A}\lambda_{1\dot{\alpha}})(M^{BF}\widetilde{\varrho}_{2F}\eta_{2\beta}) \\
&= 2ig^2(M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{2B})(\varrho_3^C\widetilde{\varrho}_{4C})\frac{\langle 3,\eta_2\rangle[1,4]}{\langle 1,2\rangle[1,2]} \\
&= 2ig^2(M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{2B})(\varrho_3^C\widetilde{\varrho}_{4C})\frac{\langle 2,3\rangle^2\langle\eta_2,3\rangle\langle 4,1\rangle}{\prod_{i=1}^{4}\langle i,i+1\rangle},
\end{aligned}
\tag{C.23}
$$

and

$$
\begin{aligned}
A_d &= (\widetilde{\varrho}_{4D}\tilde{\lambda}_4^{\dot{\delta}})(\varrho_3^C\lambda_3^{\gamma})\left(ig\sigma^{\mu}_{\gamma\dot{\delta}}\delta^{D}{}_{C}\right)\left[\frac{-i\eta_{\mu\nu}}{(p_1+p_2)^2}\right]\left(ig\sigma^{\nu}_{\alpha\dot{\beta}}\delta^{B}{}_{A}\right)(-M^{AE}\widetilde{\varrho}_{1E}\eta_1^{\alpha})(\widetilde{\varrho}_{2B}\tilde{\lambda}_2^{\dot{\beta}}) \\
&= 2ig^2(M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{2B})(\varrho_3^C\widetilde{\varrho}_{4C})\frac{\langle 3,\eta_1\rangle[2,4]}{\langle 1,2\rangle[1,2]} \\
&= -2ig^2(M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{2B})(\varrho_3^C\widetilde{\varrho}_{4C})\frac{\langle 2,3\rangle\langle 3,1\rangle\langle 3,\eta_1\rangle\langle 4,1\rangle}{\prod_{i=1}^{4}\langle i,i+1\rangle}.
\end{aligned}
\tag{C.24}
$$

The Feynman rules in figure 7 give:

$$
\begin{aligned}
A_e &= (\widetilde{\varrho}_{4D}\lambda_{4\dot{\delta}})(\widetilde{\varrho}_{1A}\lambda_1^{\dot{\alpha}})\left(2ig\Gamma^{\mathcal{I}DA}\delta_{\dot{\alpha}}{}^{\dot{\delta}}\right)\left[\frac{-i\delta_{\mathcal{I}\mathcal{J}}}{(p_1-p_4)^2}\right]\left(2ig\Gamma^{\mathcal{J}}_{CB}\delta^{\beta}{}_{\gamma}\right)(\varrho_3^C\lambda_3^{\gamma})(M^{BE}\widetilde{\varrho}_{2E}\eta_{2\beta}) \\
&= -2ig^2\frac{\langle 3,\eta_2\rangle}{\langle 1,4\rangle}\Big(M^{AB}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{2B}\varrho_3^C\widetilde{\varrho}_{4C} - M^{BD}\widetilde{\varrho}_{2B}\widetilde{\varrho}_{4D}\varrho_3^C\widetilde{\varrho}_{1C}\Big),
\end{aligned}
\tag{C.25}
$$

and

$$
\begin{aligned}
A_f &= (\varrho_3^C\lambda_3^{\gamma})(-M^{DH}\widetilde{\varrho}_{4H}\eta_{4\delta})\left(2ig\Gamma^{\mathcal{I}}_{CD}\delta^{\delta}{}_{\gamma}\right)\left[\frac{-i\delta_{\mathcal{I}\mathcal{J}}}{(p_1+p_2)^2}\right]\left(2ig\Gamma^{\mathcal{J}AB}\delta_{\dot{\beta}}{}^{\dot{\alpha}}\right)(-\widetilde{\varrho}_{1A}\tilde{\lambda}_{1\dot{\alpha}})(\widetilde{\varrho}_{2B}\tilde{\lambda}_2^{\dot{\beta}}) \\
&= -2ig^2\frac{\langle 3,\eta_4\rangle}{\langle 1,2\rangle}\Big(M^{BD}\widetilde{\varrho}_{4D}\widetilde{\varrho}_{2B}\varrho_3^C\widetilde{\varrho}_{1C} - M^{AD}\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4D}\varrho_3^C\widetilde{\varrho}_{2C}\Big)
\end{aligned}
\tag{C.26}
$$

by ithe dentity (A.13).

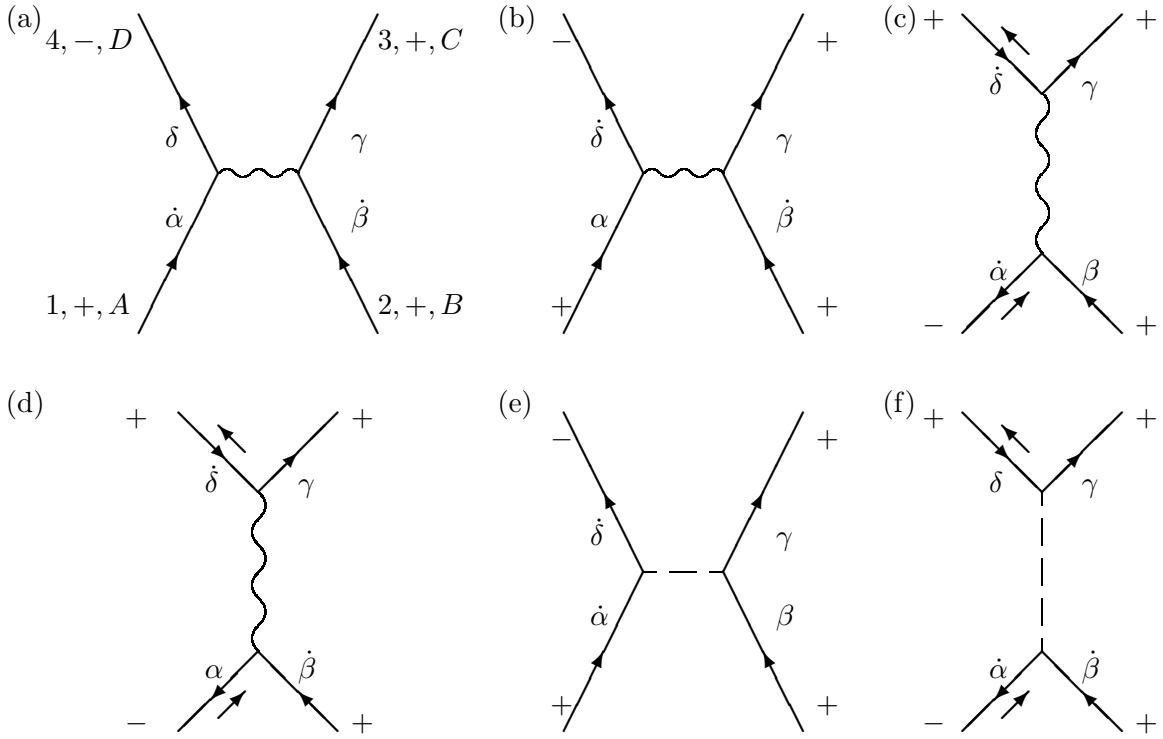

Figure 14: Planar Feynman diagrams with four external fermions corresponding to the extended MHV amplitude $A_{\mathcal{O}(M)}(+1/2,+1/2,-1/2,+1/2)$.

Altogether, the extended MHV amplitude for the four external fermions is

$$\begin{aligned} & A_{\mathcal{O}(M)}(+1/2,+1/2,-1/2,+1/2) = A_a + \cdots + A_f \\ \longrightarrow\ & \frac{2ig^2}{\prod_1^4\langle i,i+1\rangle}\Big\{\widetilde{\varrho}_{1A}M^{AB}\widetilde{\varrho}_{2B}\varrho_3^C\widetilde{\varrho}_{4C}\langle 1,2\rangle\langle 2,3\rangle\langle 3,1\rangle \\ & \quad +\widetilde{\varrho}_{1A}\varrho_3^A\widetilde{\varrho}_{2B}M^{BD}\widetilde{\varrho}_{4D}\langle 2,4\rangle\langle 2,3\rangle\langle 3,4\rangle \\ & \quad +\widetilde{\varrho}_{2B}\varrho_3^B\widetilde{\varrho}_{1A}M^{AD}\widetilde{\varrho}_{4D}\langle 4,1\rangle\langle 1,3\rangle\langle 3,4\rangle\Big\}. \end{aligned} \tag{C.27}$$

C.2.3 $A_{\mathcal{O}(M)}(+1/2,0,0,+1/2)$

The relevant Feynman diagrams are given in figure 15. Diagram (e) involves a 3-scalar vertex, which is due to the presence of the chiral mass term (as discussed in section 6 and appendix B.6).

The fermion-exchanging diagrams are calculated as usual:

$$\begin{aligned} A_a &= \varphi_{3\mathcal{J}}M^{BM}\widetilde{\varrho}_{4M}\eta_4^\beta\left(2ig\Gamma^{\mathcal{J}}_{BC}\right)\left[\frac{i(p_1+p_2)_{\beta\dot\alpha}\delta_D^C}{(p_1+p_2)^2}\right]\left(2ig\Gamma^{\mathcal{I}DA}\right)\widetilde{\varrho}_{1A}\tilde\lambda_1^{\dot\alpha}\,\varphi_{2\mathcal{I}} \\ &= -2ig^2\widetilde{\varrho}_{4C}M^{CB}\varphi_{3BD}\varphi_2^{DA}\widetilde{\varrho}_{1A}\frac{\langle 2,3\rangle\langle 3,4\rangle\langle 4,1\rangle\langle\eta_4,2\rangle}{2\prod_{i=1}^4\langle i,i+1\rangle} \\ &\longrightarrow -2ig^2\widetilde{\varrho}_{4C}M^{CB}\varphi_{3BD}\varphi_2^{DA}\widetilde{\varrho}_{1A}\frac{\langle 2,3\rangle\langle 3,4\rangle\langle 4,1\rangle}{2\prod_{i=1}^4\langle i,i+1\rangle}, \end{aligned} \tag{C.28}$$

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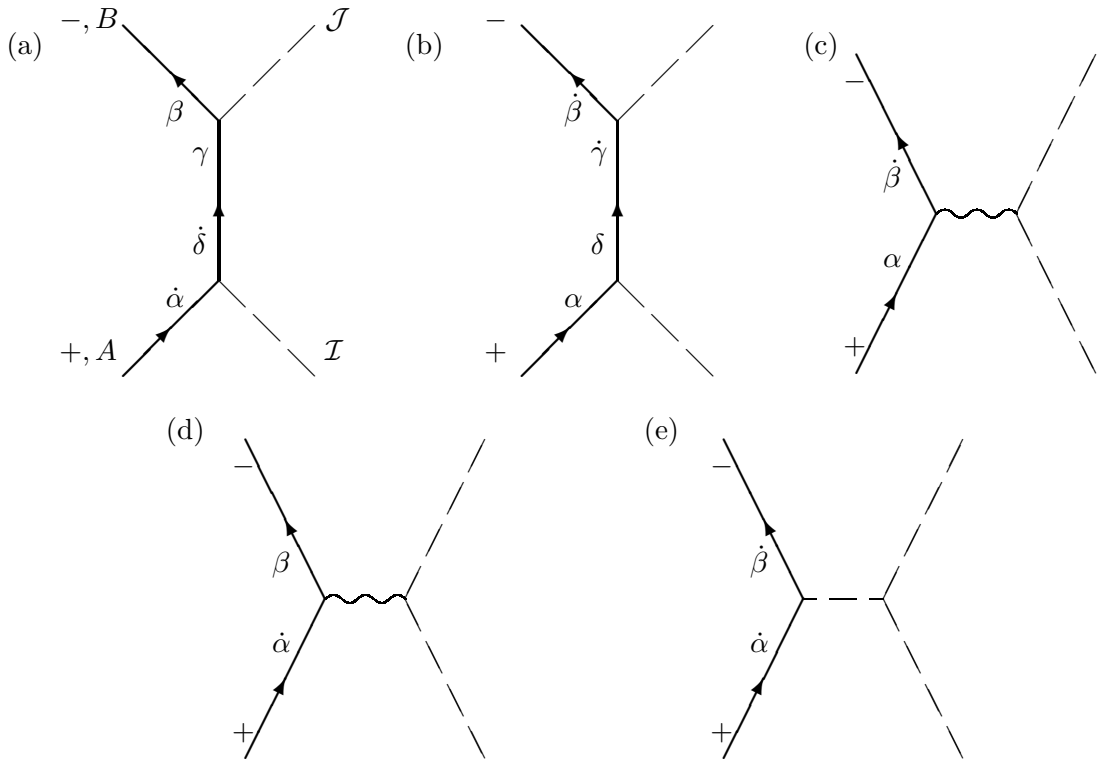

Figure 15: Planar Feynman diagrams with two external fermions and two external scalars corresponding to the extended MHV amplitude $A_{\mathcal{O}(M)}(+1/2, 0, 0, +1/2)$.

and

$$\begin{aligned} A_b &= \varphi_{3\mathcal{J}}\widetilde{\varrho}_{4B}\tilde{\lambda}_{4\dot{\beta}}\left(2ig\Gamma^{\mathcal{J}BC}\right)\left[\frac{i(p_1+p_2)^{\dot{\beta}\alpha}\delta^D_C}{(p_1+p_2)^2}\right]\left(2ig\Gamma^{\mathcal{I}DA}\right)M^{AD}\widetilde{\varrho}_{1D}\eta_{1\alpha}\,\varphi_{2\mathcal{I}} \\ &= -2ig^2\widetilde{\varrho}_{4B}\varphi_3^{BD}\varphi_{2DA}M^{AD}\widetilde{\varrho}_{1D}\frac{\langle 1,2\rangle\langle 2,3\rangle\langle 4,1\rangle\langle 3,\eta_1\rangle}{2\prod_{i=1}^4\langle i,i+1\rangle} \\ &\longrightarrow -2ig^2\widetilde{\varrho}_{4B}\varphi_3^{BD}\varphi_{2DA}M^{AD}\widetilde{\varrho}_{1D}\frac{\langle 1,2\rangle\langle 2,3\rangle\langle 4,1\rangle}{2\prod_{i=1}^4\langle i,i+1\rangle}. \end{aligned} \tag{C.29}$$

The gluon-exchanging diagrams are given by

$$\begin{aligned} A_c &= \varphi_{3\mathcal{J}}\widetilde{\varrho}_{4B}\tilde{\lambda}_{4\dot{\beta}}\left(ig\bar{\sigma}^{\mu\dot{\beta}\alpha}\delta^B_A\right)M^{AC}\widetilde{\varrho}_{1C}\eta_{1\alpha}\left[\frac{-i\eta_{\mu\nu}}{(p_4-p_1)^2}\right]\left(\frac{ig}{2}(p_2+p_3)^\nu\delta^{\mathcal{I}\mathcal{J}}\right)\varphi_{2\mathcal{I}} \\ &= ig^2\widetilde{\varrho}_{4B}M^{BC}\widetilde{\varrho}_{1C}\varphi_{1A'B'}\varphi_2^{A'B'}\frac{\langle 1,2\rangle\langle 3,4\rangle(\langle 1,3\rangle\langle\eta_1,2\rangle+\langle 1,2\rangle\langle\eta_1,3\rangle)}{2\prod_{i=1}^4\langle i,i+1\rangle} \\ &\longrightarrow -ig^2\widetilde{\varrho}_{4B}M^{BC}\widetilde{\varrho}_{1C}\varphi_{1A'B'}\varphi_2^{A'B'}\frac{\langle 1,2\rangle\langle 3,4\rangle(\langle 1,3\rangle+\langle 1,2\rangle)}{2\prod_{i=1}^4\langle i,i+1\rangle} \end{aligned} \tag{C.30}$$

and

$$\begin{aligned} A_d &= \varphi_{3\mathcal{J}}M^{BD}\widetilde{\varrho}_{4D}\eta_4^\beta\left(ig\sigma^\mu_{\beta\dot{\alpha}}\delta^A_B\right)\widetilde{\varrho}_{1A}\tilde{\lambda}_1^{\dot{\alpha}}\left[\frac{-i\eta_{\mu\nu}}{(p_4+p_1)^2}\right]\left(\frac{ig}{2}(p_2+p_3)^\nu\delta^{\mathcal{I}\mathcal{J}}\right)\varphi_{2\mathcal{I}} \\ &= ig^2\widetilde{\varrho}_{4B}M^{BC}\widetilde{\varrho}_{1C}\varphi_{1A'B'}\varphi_2^{A'B'}\frac{\langle 1,2\rangle\langle 3,4\rangle(\langle 2,4\rangle\langle 3,\eta_4\rangle-\langle 2,\eta_4\rangle\langle 3,4\rangle)}{2\prod_{i=1}^4\langle i,i+1\rangle} \end{aligned}$$

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$$\longrightarrow -ig^2\widetilde{\varrho}_{4B}M^{BC}\widetilde{\varrho}_{1C}\varphi_{1A'B'}\varphi_2^{A'B'}\frac{\langle 1,2\rangle\langle 3,4\rangle(\langle 2,4\rangle+\langle 3,4\rangle)}{2\prod_{i=1}^4\langle i,i+1\rangle}. \tag{C.31}$$

Finally, the Feynman rule for the 3-scalar vertex as depicted in figure 4 gives

$$\begin{aligned}
A_e &= \widetilde{\varrho}_{4B}\tilde{\lambda}_{4\dot\beta}\widetilde{\varrho}_{1A}\tilde{\lambda}_1^{\dot\alpha}\left(2ig\Gamma^{\mathcal{L}BA}\delta_{\dot\alpha}^{\dot\beta}\right)\left[\frac{-i\delta_{\mathcal{L}\mathcal{K}}}{(p_1-p_4)^2}\right]\Big(-\frac{ig}{2}M^{CD}\Gamma^{\mathcal{K}}_{CE}\Gamma^{\mathcal{I}}_{DF}\Gamma^{\mathcal{J}EF}\Big)\,\varphi_{3\mathcal{J}}\,\varphi_{2\mathcal{I}}\\
&= -2ig^2\frac{\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}}{\langle 1,4\rangle[1,4]}[4,1]\Big(\delta_{\mathcal{L}\mathcal{K}}\Gamma^{\mathcal{L}BA}\Gamma^{\mathcal{K}}_{CE}\Big)M^{CD}\varphi_{2DF}\,\varphi_3^{EF}\\
&= -ig^2\frac{\langle 1,2\rangle\langle 2,3\rangle\langle 3,4\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}\,\widetilde{\varrho}_{1A}\widetilde{\varrho}_{4B}\Big(M^{BC}\varphi_{2CD}\,\varphi_3^{AD}-M^{AC}\varphi_{2CD}\,\varphi_3^{BD}\Big)\\
&= -ig^2\frac{\langle 1,2\rangle\langle 2,3\rangle\langle 3,4\rangle}{\prod_{i=1}^4\langle i,i+1\rangle}\Big\{\widetilde{\varrho}_{4C}M^{CB}\varphi_{3BD}\varphi_2^{DA}\widetilde{\varrho}_{1A}+\widetilde{\varrho}_{4B}\varphi_3^{BD}\varphi_{2DA}M^{AD}\widetilde{\varrho}_{1D}\\
&\qquad\qquad +\frac{1}{2}\widetilde{\varrho}_{4C}M^{CB}\widetilde{\varrho}_{1B}\varphi_3^{B'D'}\varphi_{2D'B'}\Big\},
\end{aligned}\tag{C.32}$$

where the identity (A.18) is used in the last line.

Put all together, we have

$$\begin{aligned}
&A_{\mathcal{O}(M)}(+1/2,0,0,+1/2)=A_a+A_b+A_c+A_d+A_e\\
\longrightarrow\ &\frac{ig^2}{\prod_{i=1}^4\langle i,i+1\rangle}\Big\{(\langle 2,3\rangle\langle 3,4\rangle\langle 1,4\rangle+\langle 2,3\rangle\langle 3,4\rangle\langle 2,1\rangle)\widetilde{\varrho}_{4C}M^{CB}\varphi_{3BD}\varphi_2^{DA}\widetilde{\varrho}_{1A}\\
&\qquad +(\langle 1,2\rangle\langle 2,3\rangle\langle 1,4\rangle+\langle 2,3\rangle\langle 3,4\rangle\langle 2,1\rangle)\widetilde{\varrho}_{4B}\varphi_3^{BD}\varphi_{2DA}M^{AD}\widetilde{\varrho}_{1D}\\
&\qquad \frac{1}{2}(\langle 1,2\rangle\langle 1,4\rangle\langle 3,4\rangle+\langle 2,3\rangle\langle 3,4\rangle\langle 2,1\rangle)\widetilde{\varrho}_{4B}M^{BC}\widetilde{\varrho}_{1C}\varphi_3^{B'D'}\varphi_{2B'D'}\Big\}\\
=\ &\frac{ig^2}{\prod_{i=1}^4\langle i,i+1\rangle}\Big\{\langle 2,3\rangle\langle 3,4\rangle\langle 4,2\rangle\widetilde{\varrho}_{4C}M^{CB}\varphi_{3BD}\varphi_2^{DA}\widetilde{\varrho}_{1A}\\
&\qquad +\langle 1,2\rangle\langle 2,3\rangle\langle 3,1\rangle\widetilde{\varrho}_{4B}\varphi_3^{BD}\varphi_{2DA}M^{AD}\widetilde{\varrho}_{1D}\\
&\qquad \frac{1}{2}(\langle 1,2\rangle\langle 2,3\rangle\langle 3,4\rangle+\langle 1,2\rangle\langle 1,4\rangle\langle 3,4\rangle)\widetilde{\varrho}_{4B}M^{BC}\widetilde{\varrho}_{1C}\varphi_3^{B'D'}\varphi_{2B'D'}\Big\}.
\end{aligned}\tag{C.33}$$

References

[1] D.-W. Chiou, O.J. Ganor, Y.P. Hong, B.S. Kim and I. Mitra, *Massless and massive three dimensional super Yang-Mills theory and mini-twistor string theory*, *Phys. Rev.* **D 71** (2005) 125016 [hep-th/0502076].

[2] R. Penrose, *Twistor algebra*, *J. Math. Phys.* **8** (1967) 345.

[3] E. Witten, *Perturbative gauge theory as a string theory in twistor space*, *Commun. Math. Phys.* **252** (2004) 189 [hep-th/0312171].

[4] S. Sethi, *Supermanifolds, rigid manifolds and mirror symmetry*, *Nucl. Phys.* **B 430** (1994) 31 [hep-th/9404186].

[5] M. Aganagic and C. Vafa, *Mirror symmetry and supermanifolds*, hep-th/0403192.

[6] N. Berkovits, *An alternative string theory in twistor space for $N = 4$ super-Yang-Mills*, *Phys. Rev. Lett.* **93** (2004) 011601 [hep-th/0402045].

JHEP03(2006)027

[7] N. Berkovits and L. Motl, *Cubic twistorial string field theory*, *JHEP* **04** (2004) 056 [hep-th/0403187].

[8] W. Siegel, *Untwisting the twistor superstring*, hep-th/0404255.

[9] F. Cachazo and P. Svrcek, *Lectures on twistor strings and perturbative Yang-Mills theory*, *PoS* **RTN2005** (2005) 004 [hep-th/0504194].

[10] S.D. Badger, E.W.N. Glover, V.V. Khoze and P. Svrcek, *Recursion relations for gauge theory amplitudes with massive particles*, *JHEP* **07** (2005) 025 [hep-th/0504159].

[11] P.S. Howe, K.S. Stelle and P.K. Townsend, *Superactions*, *Nucl. Phys.* **B 191** (1981) 445.

[12] K.A. Intriligator, *Bonus symmetries of $N = 4$ super-Yang-Mills correlation functions via AdS duality*, *Nucl. Phys.* **B 551** (1999) 575 [hep-th/9811047].

[13] N. Berkovits and E. Witten, *Conformal supergravity in twistor-string theory*, *JHEP* **08** (2004) 009 [hep-th/0406051].

[14] C.-h. Ahn, *Comments on MHV tree amplitudes for conformal supergravitons from topological b-model*, *JHEP* **07** (2005) 004 [hep-th/0504109].

[15] S.J. Parke and T.R. Taylor, *An amplitude for N gluon scattering*, *Phys. Rev. Lett.* **56** (1986) 2459.

[16] F.A. Berends and W.T. Giele, *Recursive calculations for processes with N gluons*, *Nucl. Phys.* **B 306** (1988) 759.

[17] Z. Bern, L.J. Dixon and D.A. Kosower, *Progress in one-loop QCD computations*, *Ann. Rev. Nucl. Part. Sci.* **46** (1996) 109–148 [hep-ph/9602280].

[18] G. Georgiou and V.V. Khoze, *Tree amplitudes in gauge theory as scalar MHV diagrams*, *JHEP* **05** (2004) 070 [hep-th/0404072].

[19] J.B. Wu and C.J. Zhu, *MHV vertices and fermionic scattering amplitudes in gauge theory with quarks and gluinos*, *JHEP* **09** (2004) 063 [hep-th/0406146].

[20] A.D. Popov and C. Sämann, *On supertwistors, the Penrose-Ward transform and $N = 4$ super Yang-Mills theory*, hep-th/0405123.

[21] A.D. Popov and M. Wolf, *Topological B-model on weighted projective spaces and self-dual models in four dimensions*, *JHEP* **09** (2004) 007 [hep-th/0406224].

[22] C. Sämann, *The topological B-model on fattened complex manifolds and subsectors of $N = 4$ self-dual Yang-Mills theory*, *JHEP* **01** (2005) 042 [hep-th/0410292].

[23] M. Wolf, *On hidden symmetries of a super gauge theory and twistor string theory*, *JHEP* **02** (2005) 018 [hep-th/0412163]; *Twistors and aspects of integrability of self-dual SYM theory*, hep-th/0511230.

[24] R. Dijkgraaf, M.T. Grisaru, C.S. Lam, C. Vafa and D. Zanon, *Perturbative computation of glueball superpotentials*, *Phys. Lett.* **B 573** (2003) 138 [hep-th/0211017].

[25] N. Nekrasov, H. Ooguri and C. Vafa, *S-duality and topological strings*, *JHEP* **10** (2004) 009 [hep-th/0403167].

[26] M. Aganagic, R. Dijkgraaf, A. Klemm, M. Marino and C. Vafa, *Topological strings and integrable hierarchies*, *Commun. Math. Phys.* **261** (2006) 451 [hep-th/0312085].

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[27] H. Ooguri, A. Strominger and C. Vafa, *Black hole attractors and the topological string*, *Phys. Rev.* **D 70** (2004) 106007 [hep-th/0405146].

[28] R. Dijkgraaf, S. Gukov, A. Neitzke and C. Vafa, *Topological M-theory as unification of form theories of gravity*, hep-th/0411073.

[29] M. Aganagic, H. Ooguri, N. Saulina and C. Vafa, *Black holes, q-deformed 2D Yang-Mills and non-perturbative topological strings*, *Nucl. Phys.* **B 715** (2005) 304 [hep-th/0411280].

[30] N. Nekrasov, *A la recherche de la m-theorie perdue. Z theory: casing M/F theory*, hep-th/0412021.

[31] N. Nekrasov, *Z-theory: Chasing image theory*, *Comptes Rendus Physique* **6** (2005) 261.

[32] M. Kulaxizi and K. Zoubos, *Marginal deformations of $N = 4$ SYM from open/closed twistor strings*, *Nucl. Phys.* **B 738** (2006) 317 [hep-th/0410122].

[33] C.H. Ahn, *$N = 1$ conformal supergravity and twistor-string theory*, *JHEP* **10** (2004) 064 [hep-th/0409195]; *$N = 2$ conformal supergravity from twistor-string theory*, hep-th/0412202.

[34] J. Park and S.J. Rey, *Supertwistor orbifolds: gauge theory amplitudes and topological strings*, *JHEP* **12** (2004) 017 [hep-th/0411123].

[35] S. Giombi et al., *Orbifolding the twistor string*, *Nucl. Phys.* **B 719** (2005) 234 [hep-th/0411171].

[36] R.G. Leigh and M.J. Strassler, *Exactly marginal operators and duality in four-dimensional $N = 1$ supersymmetric gauge theory*, *Nucl. Phys.* **B 447** (1995) 95 [hep-th/9503121].

[37] O. Aharony, B. Kol and S. Yankielowicz, *On exactly marginal deformations of $N = 4$ sym and type-IIB supergravity on $AdS_5 \times S^5$*, *JHEP* **06** (2002) 039 [hep-th/0205090].

[38] N. Dorey, *S-duality, deconstruction and confinement for a marginal deformation of $N = 4$ SUSY Yang-Mills*, *JHEP* **08** (2004) 043 [hep-th/0310117].

[39] N. Dorey and T.J. Hollowood, *On the coulomb branch of a marginal deformation of $N = 4$ SUSY Yang-Mills*, *JHEP* **06** (2005) 036 [hep-th/0411163].

[40] O. Lunin and J. Maldacena, *Deforming field theories with U(1) $\times$ U(1) global symmetry and their gravity duals*, *JHEP* **05** (2005) 033 [hep-th/0502086].

[41] A.D. Popov, C. Sämann and M. Wolf, *The topological B-model on a mini-supertwistor space and supersymmetric bogomolny monopole equations*, *JHEP* **10** (2005) 058 [hep-th/0505161].

[42] C. Sämann, *On the mini-superambitwistor space and $N = 8$ super Yang-Mills theory*, hep-th/0508137; *The mini-superambitwistor space*, hep-th/0511251.

[43] N.J. Hitchin, *Monopoles and geodesics*, *Commun. Math. Phys.* **83** (1982) 579.

[44] M. F. Atiyah and N. Hitchin, *The geometry and dynamics of magnetic monopoles* Princeton University Press, 1988.

[45] O. Lechtenfeld and C. Sämann, *Matrix models and D-branes in twistor string theory*, hep-th/0511130.

[46] L. Dixon, *Calculating scattering amplitudes efficiently*, hep-th/9601359.

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